# A RATIONAL APPROACH TO THE DESIGN OF BITUMINOUS PAVING MIXTURES

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## INTRODUCTION

It should be a matter of some concern to thoughtful airport and highway engineers that, while our present approach to rigid pavement design is entirely rational throughout, that is, on a pounds per square inch or other unit strength basis, current flexible pavement design is largely empirical.

For rigid pavement design, the thickness of slab required can be obtained from equations developed by Westergaard or Hogg. Furthermore, there are well established principles for designing portland cement concrete mixtures of any desired compressive or flexural strength on a p.s.i. basis.

For flexible pavement design, on the other hand, both the overall thickness required and the methods to be employed to determine that thickness are highly controversial at the present time. As for designing bituminous mixtures of any specified strength, the stability tests in most common use, Hubbard-Field, Marshall, and Hveem Stabilometer, are entirely incapable of measuring the strength of these mixtures in terms of shear or any other fundamental property on a p.s.i. basis. They are strictly empirical tests.

Empirical methods have a number of very serious disadvantages. With them it is difficult to avoid either overdesign or underdesign. It is dangerous to extrapolate their results to cover conditions beyond those under which they were established. Most serious of all insofar as the design of bituminous mixtures is concerned, empirical tests may completely overlook, or be unable to take into account, fundamental factors that have an important bearing on the stability developed by bituminous pavements in service.

In every engineering field, rational methods of design, in which the strengths of all materials employed are utilized on a p.s.i. or other unit strength basis, should be an ultimate objective. In addition to the three methods already named, the Texas Punching Shear, Florida Bearing Value, Modified Hubbard-Field, Campen's Bearing Index, Unconfined Compression, Beam, Impact,

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Tensile, and Indentation Tests, etc., have been developed in recent years. Most of these are empirical tests. The fact that the number of different empirical methods for measuring the stability of bituminous mixtures continues to increase provides the most tangible proof of the current dissatisfaction of highway and airport engineers with the inadequacies of these tests. This search for a satisfactory stability test for bituminous mixtures is not likely to end until a generally acceptable fundamental approach to the measurement of the stability of bituminous pavements in service is developed.

There are some who feel that from the very nature of the material, it is not possible to design the stability of bituminous pavements on a unit strength basis. However, the fact that most of them carry heavy wheel loads for years without any sign of instability is proof that bituminous pavements do possess definite strength that must be capable of measurement on a p.s.i. or other unit strength basis by means of an appropriate test.

Canada's Department of Transport has been interested in the development of a rational method of design for bituminous paving mixtures, partly because of the confusion that exists at the present time concerning the actual significance of the various tests such as Hubbard-Field, Marshall, Hveem, Stabilometer, etc., employed to measure their stability, and partly because it has been suggested that large future aircraft may be equipped with landing wheels carrying tire pressures of 300 to 400 p.s.i. The Department of Transport would like to know whether or not dense, durable, workable bituminous mixtures can be designed to carry heavy wheel loads under such high tire pressures.

The experience of fifty years has indicated how bituminous mixtures must be designed to have adequate workability, density, durability, etc. This paper is confined, therefore, to the description of a method for measuring the strength of bituminous mixtures on a p.s.i. basis after they have met the necessary requirements in these other respects, and to a discussion of its utilization for the determination of the stability of bituminous pavements in the field.

Of all the methods currently available for this purpose, the triaxial test appears to be the most promising. An outline of the manner in which the data provided by this test can apparently be utilized to design bituminous pavements of any required strength on a p.s.i. basis, is the principal objective of this paper.

Papers dealing with the triaxial test have already appeared, or are being currently published, in the Proceedings of this Association, notably by Endersby[1], Nijboer[2], Smith[3], Goetz and Chen[4], and Housel[25]. Hveem[6], Terzaghi[7], Casagrande[8], Taylor[9], Rutledge[10] and Holtz[11], are other well known names that any reference to this test brings to mind.

Nijboer[2] and Smith[3] have developed methods for utilizing data from the triaxial test for the design of bituminous paving mixtures on a p.s.i. basis. Nijboer makes use of the Prandtl equation for this purpose. Smith's development is derived from the mathematical theory of elasticity, and is, therefore, subject to whatever uncertainties may result from the application of this theory to stressed materials close to the loaded area.

As will be pointed out in this paper, it would appear that other factors, in addition to those included in the methods of Smith and Nijboer, may have considerable influence on the stability of bituminous pavements in service. A knowledge of the effect of each of these factors on pavement stability is important, since it may lead to a much wider selection of materials, to more economical pavements, and to a better understanding of the fundamental requirements of bituminous pavement design.

Before proceeding to a discussion of the triaxial test and its application to bituminous pavement design, brief mention should be made of the three principal conditions of pavement stability that must be considered. These are:

(a) Stability under the wheel loads of stationary vehicles.
(b) Stability under the wheel loads of vehicles moving at a relatively high and reasonably uniform rate of speed.
(c) Stability under the braking and accelerating stresses of traffic.

When pavements are to be subjected to two or to all three of these types of load, it is necessary to determine which of the loading conditions is most severe from the point of view of pavement stability. The stability of any bituminous paving mixture should be designed for the most critical condition of load to which it is likely to be exposed for a period of time during its useful life.

## THE TRIAXIAL TEST

The general nature of the problem, with which this paper is concerned, can be more easily visualized by reference to Figure 1, which is a diagram of possible surfaces of shearing failure under a loaded area on a flexible pavement on an airport or highway. The overall design problem from the standpoint of stability consists of preventing detrimental shear within any one of the three elements of the composite structure, the subgrade, the base

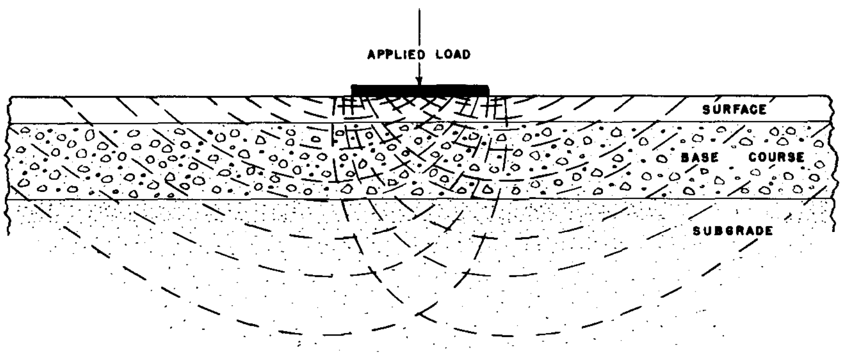



Figure 1. Diagram of Shear Planes Under a Loaded Area

course, and the wearing surface. If sufficient plastic shear develops in any one or more of these three elements, rutting and upheaval of the pavement surface will occur.

Detrimental plastic shear of the subgrade is prevented by an adequate overall thickness of base course and wearing surface[12,13,14]. Serious plastic shear of the base course and bituminous pavement can be avoided, only if the materials selected for these two layers have adeqaute shearing resistance to the stresses of the applied loads.

For this paper, it is assumed that an adequate thickness of base and surface have been provided to prevent subgrade failure, and that the base course material itself will not fail under the shear stresses imposed by the loads applied. The fundamental problem to be investigated, therefore, is the design of bituminous paving mixtures having sufficient strength or stability in terms of pounds per square inch, to support without failure, the wheel loads and tire pressures to which they are to be subjected. The development that follows attempts to provide a rational answer to this problem on the basis of information provided by the triaxial test and the Mohr diagram.

The triaxial differs from an ordinary compression test in that provision is made for the application of controlled or measured lateral support to the specimen, while it is being subjected to vertical load. The triaxial equipment most commonly used on this continent is illustrated in Figure 2. It is sometimes referred to as the open-type, because the lateral support is maintained constant throughout the test on any one specimen. To the lucite cylinder, the two metal end pieces are fitted by means of water-tight

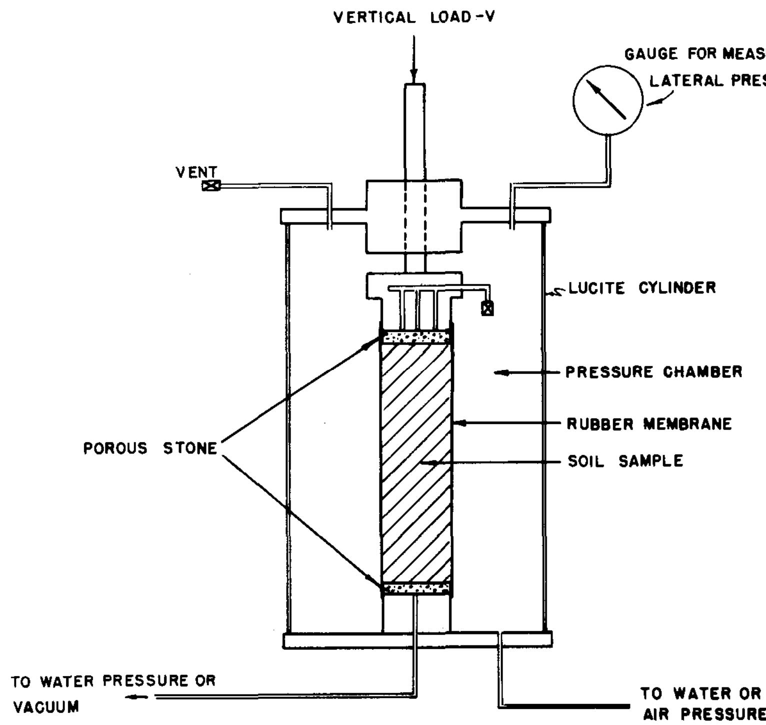



Figure 2. Sketch of Apparatus for Triaxial Compression Test

and air-tight gasketed joints. A cylindrical specimen of the material to be tested is inserted in a rubber sleeve. Porous stones at the top and bottom of the specimen may or may not be required, depending upon the material to be tested and the nature of the test data desired. By means of connections through the porous stone, the specimen within the rubber sleeve can be subjected to vacuum or water pressure, or to free drainage or no drainage, as required. Air, water, or other fluid can be pumped into the lucite cylinder to provide the magnitude of lateral support specified for the testing of each specimen. The rubber sleeve prevents the fluid within the lucite cylinder from entering the sample under test. Each specimen is subjected to a constant lateral pressure throughout the test, and increasing vertical load is applied in a standard manner until it fails. A complete triaxial test on a given material with this apparatus, usually consists of loading three or four cylindrical specimens of the material to failure, employing a different degree of lateral support for each, e.g. 0, 15, 30, and 60 p.s.i.

The data obtained from testing a given material in triaxial compression are plotted in the form of a Mohr diagram, Figure 3.

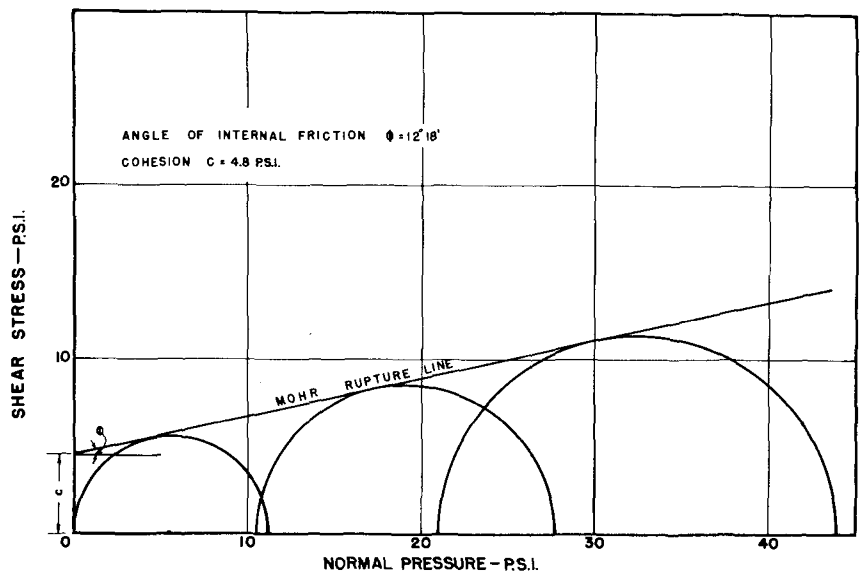



Figure 3. Typical Mohr Diagram for Triaxial Compression Test

For each specimen, the applied lateral pressure L, and the corresponding vertical pressure V that caused failure, are marked off on the horizontal axis (abscissa). Using the difference between the vertical and lateral pressure, V-L, for each specimen, as the diameter, semi-circles, known as Mohr circles, are described as shown. The tangent common to the Mohr circles is drawn, and produced to intersect the vertical axis (ordinate). The intercept on the vertical axis is designated cohesion c, while the angle between the common tangent and the horizontal is the angle of internal friction Ø. Both c and Ø are from the Coulomb equation, $s = c + n \tan Ø$.

The common tangent is generally known as the Mohr rupture line, or Mohr envelope. The Mohr envelopes for different materials have a wide range of values for both c and Ø.

Figure 4 indicates that all semi-circles that are tangent to, or below the Mohr envelope for a given material, represent equilibrium, (circle 2), or stable, (circle 3), relationships respectively, between corresponding values of lateral support L and vertical

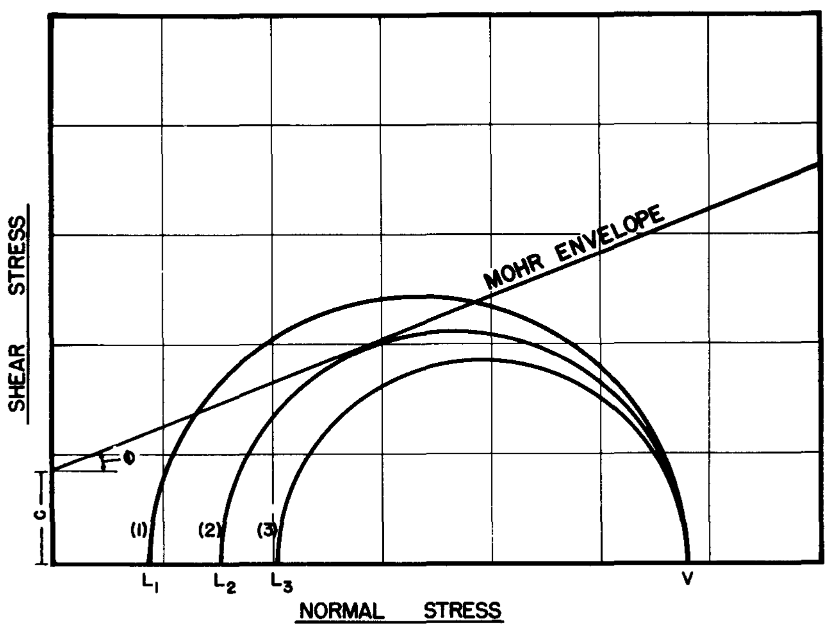



Figure 4. Mohr Circles Representing Unstable, Equilibrium, and Stable Combinations of V and L Values for A Given Material Under Stress

pressure V. Any semi-circle, (circle 1), which cuts through the Mohr envelope, indicates corresponding combinations of lateral support L and vertical pressure V, that would cause failure of this material.

For the development which follows, it is assumed that the Mohr envelope is a straight line. Whether or not this assumption is justified in the case of materials like clay soils, seems to depend upon the conditions of testing and the method employed for interpreting the results[11]. It has been reasonably well established for purely granular soils[10], and the work of Nijboer[2] and Smith[3] indicates that a straight line Mohr envelope can ordinarily be expected for properly designed bituminous mixtures.

Depending upon the position of the Mohr envelope that results from testing them, cohesive and granular materials can be conventionally divided into three groups.

(a) Purely cohesive materials, i.e. those for which the angle of internal friction $\emptyset$ is zero, but the cohesion c has a positive value, and the Mohr envelope, therefore, is parallel to the abscissa, Figure 5. Saturated clays in the quick

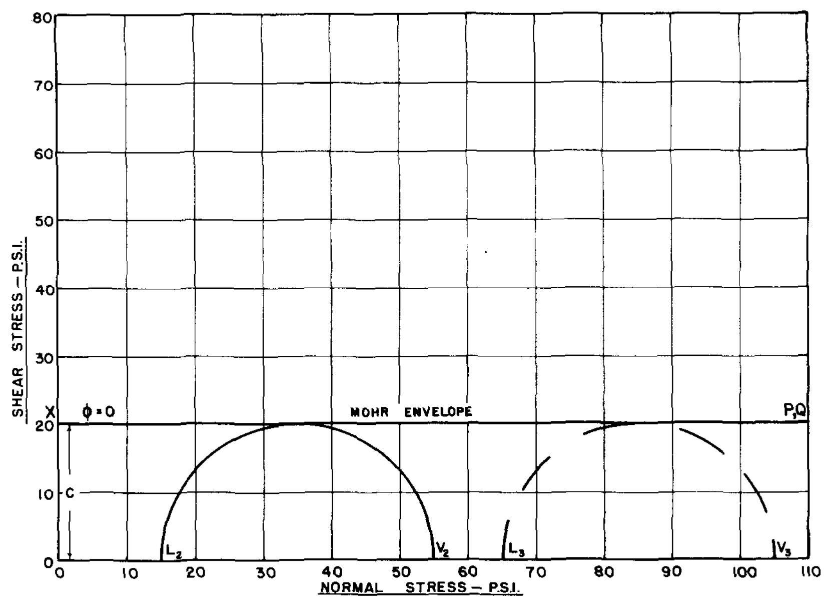



Figure 5. Mohr Diagram for Materials Having Zero Angle of Internal Friction in Triaxial Compression

triaxial test approximate these requirements[10], and bituminous mixtures with voids approximately filled or overfilled with bituminous binder, are probably other examples.

(b) Purely granular materials, i.e. those for which the cohesion c is zero, but the angle of internal friction $\emptyset$ has a positive value, and the Mohr envelope passes through the origin, Figure 6. The clean sands, gravels, crushed stone, and similar granular materials employed as the aggregates for bituminous mixtures approach these requirements.

(c) Materials which have both granular and cohesive properties, i.e. those with positive values for both cohesion c and angle of internal friction $\emptyset$, and with Mohr envelopes of positive slope and making positive intercepts with the

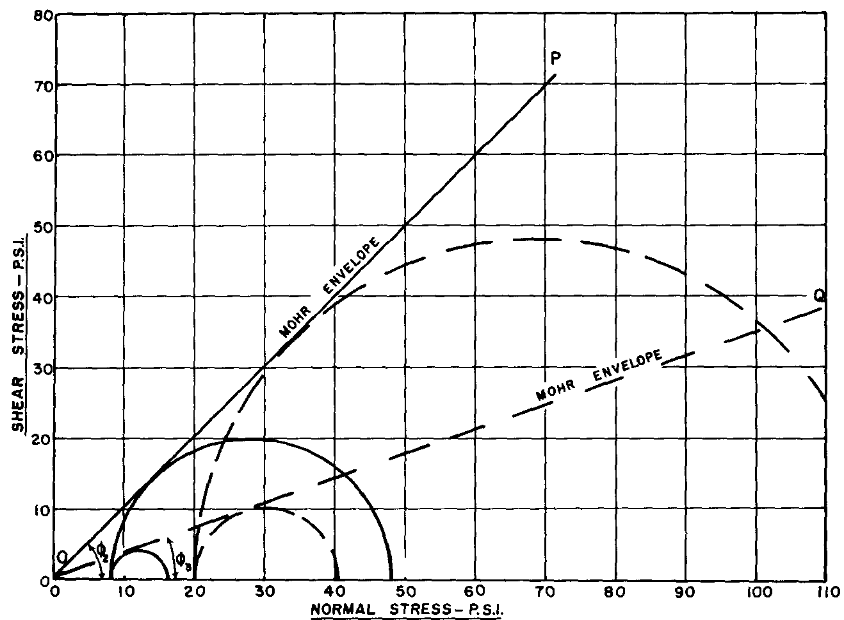



Figure 6. Mohr Diagram for Materials Having Zero Cohesion in Triaxial Compression

ordinate axis, Fig. 3. Bituminous paving mixtures[2], and remolded clays[11], are examples of materials with this type of Mohr envelope.

The Mohr diagram provides a fundamental basis for defining the term "stability" as applied to granular and cohesive materials in general, and to bituminous mixtures in particular. If several different bituminous mixtures were formed under standard conditions into cylinders of the same size, (e.g. 6-in. dia. by 12-in. high), the same magnitude of lateral support L provided for each cylinder, and the vertical load V (applied under standard conditions) at which each failed was determined, it would be generally agreed that the most stable mixture was the one that carried the greatest vertical load V at failure. Consequently, for any specified value of lateral support L, the most stable material is that for which the value of V-L is the greatest at failure. That is, the stability of a material under load is measured by the quantity V-L, where L is the amount of lateral support provided, and V is the maximum vertical load it can carry without failure.

This definition of stability is illustrated by Figure 7, in which Mohr envelopes ab, cd, and ef are given for three different materials. At lateral support $L_1$, it is apparent that the material represented by Mohr envelope ef is the most stable of the three, since $V_3$-$L_1$ is greater than $V_2$-$L_1$ for Mohr envelope cd, and than $V_1$-$L_1$ for Mohr envelope ab. On the other hand, at lateral support $L_2$, the stability rating of the three materials is exactly

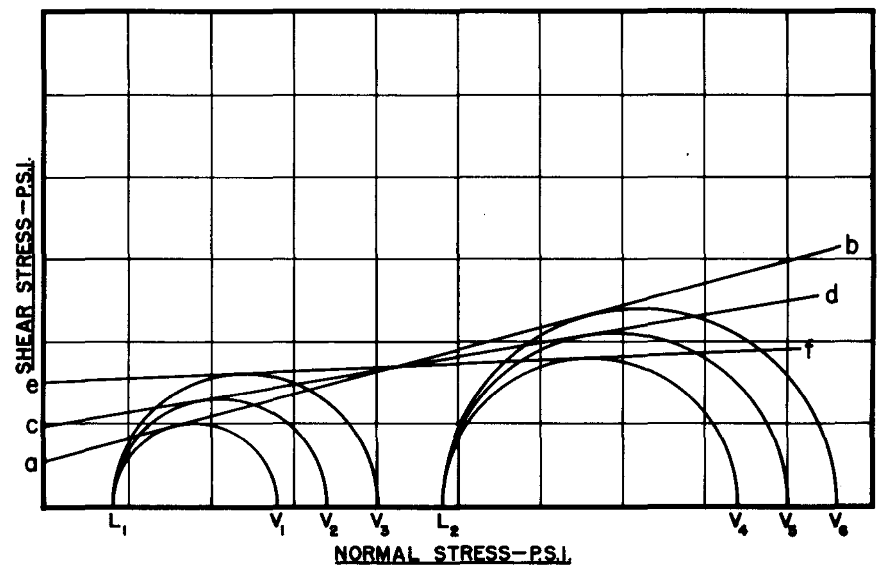



Figure 7. Illustrating the Definition of Stability

reversed, with the material represented by Mohr envelope ab being the most stable, since $V_6$-$L_2$ is greater than $V_5$-$L_2$ for Mohr envelope cd, and than $V_4$-$L_2$ for Mohr envelope ef. Figure 7, therefore, serves to emphasize the fact that the stability rating, V-L, for each of a group of materials, depends upon the magnitude of the lateral support L at which the stability determinations are made.

## DERIVATION AND APPLICATION OF AN EQUATION OF STABILITY

Figure 8 illustrates the geometrical and trigonometrical relationships required for the development of an equation of stability for materials with both granular and cohesive properties. From

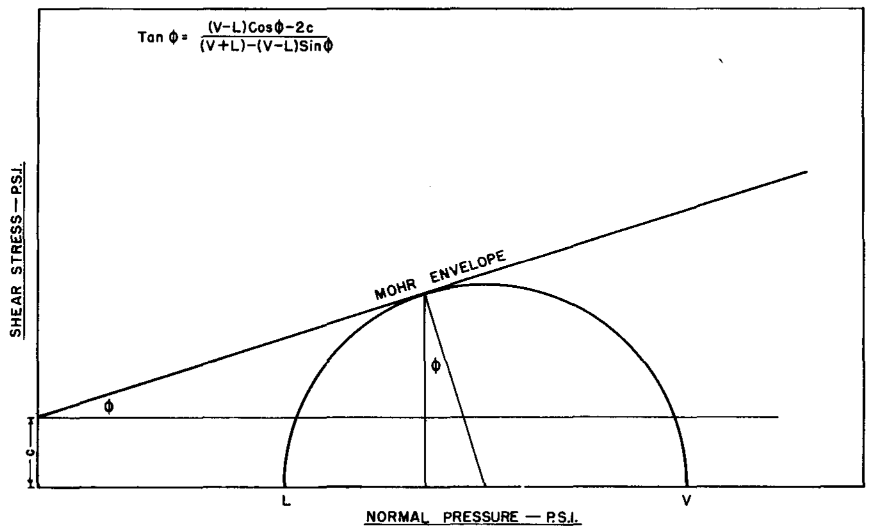



Figure 8. Trigonometrical Relationships for Mohr Diagram for Materials Having Positive Values of c and $\phi$ in Triaxial Compression

Figure 8 it is clear that the magnitude of the stability value, V-L, depends upon the magnitude of the lateral support L, the cohesion c, and the angle of internal friction $\phi$.

From Figure 8 it follows that

$$\tan \phi = \frac{\frac{V-L}{2} \cos \phi - c}{L + \frac{V-L}{2} - \frac{V-L}{2} \sin \phi} \tag{1}$$

which can be easily worked through to

$$V-L = \frac{2L \sin \phi}{1 - \sin \phi} + 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \tag{2}$$

Equation (2) is an equation of stability for materials with both granular and cohesive properties.

The stability diagram of Figure 9 is obtained when equation (2) is plotted in terms of given values of stability V-L, for different

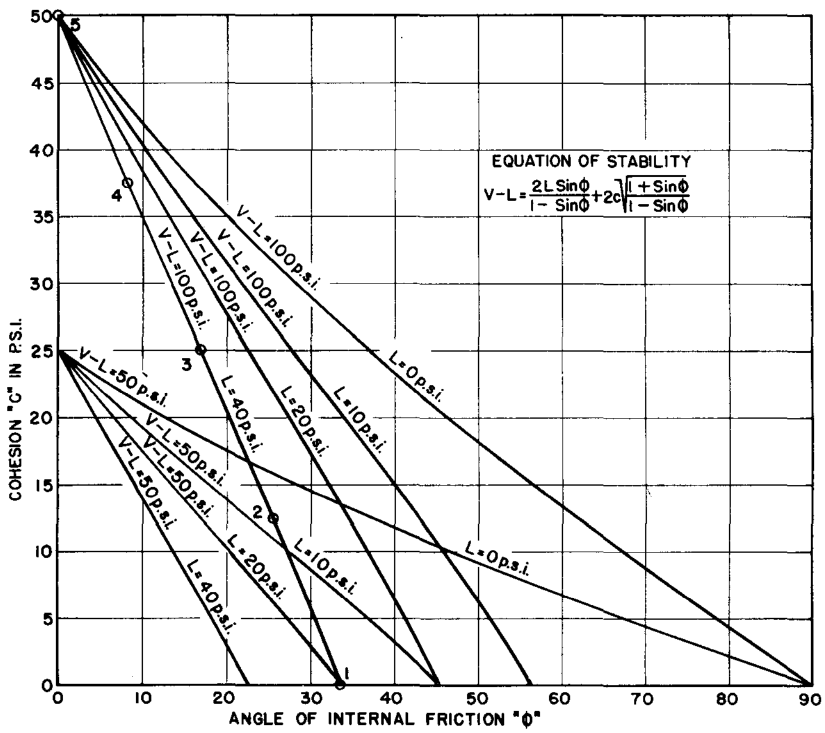



Figure 9. Stability Diagram in Terms of c, $\phi$, L and (V-L) for Materials Having Positive Values for c and $\phi$ in Triaxial Compression

degrees of lateral support L, and for various magnitudes of c and $\phi$. Each stability curve, V-L, shown in Figure 9, indicates that only those materials with combinations of c and $\phi$ that lie on or to the right of the curve, would have the stability required for the combination of vertical load V, and lateral support L, specified for that stability curve.

The stability curves of Figure 9 have the disadvantage that while V-L may be constant for a given set of curves, the corresponding value of V changes with each change in L. For example, for V-L = 100 p.s.i., when L = 10 p.s.i., the corresponding value cf V = 90 p.s.i., but for V-L = 100 p.s.i., when L = 40 p.s.i., the corresponding value of V = 60 p.s.i. That is, the value of V is different for each stability curve. It might be more useful, therefore, if stability curves somewhat similar to those of Figure 9 could be drawn for any given constant value of V, e.g. V = 100 p.s.i., as the value of L is varied.

Equation (2) can be rearranged as follows:

$$V = L\,\frac{1 + \sin \phi}{1 - \sin \phi} + 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \qquad (3)$$

Equation (3) is also a stability equation for materials with both granular and cohesive properties. This can be readily seen from

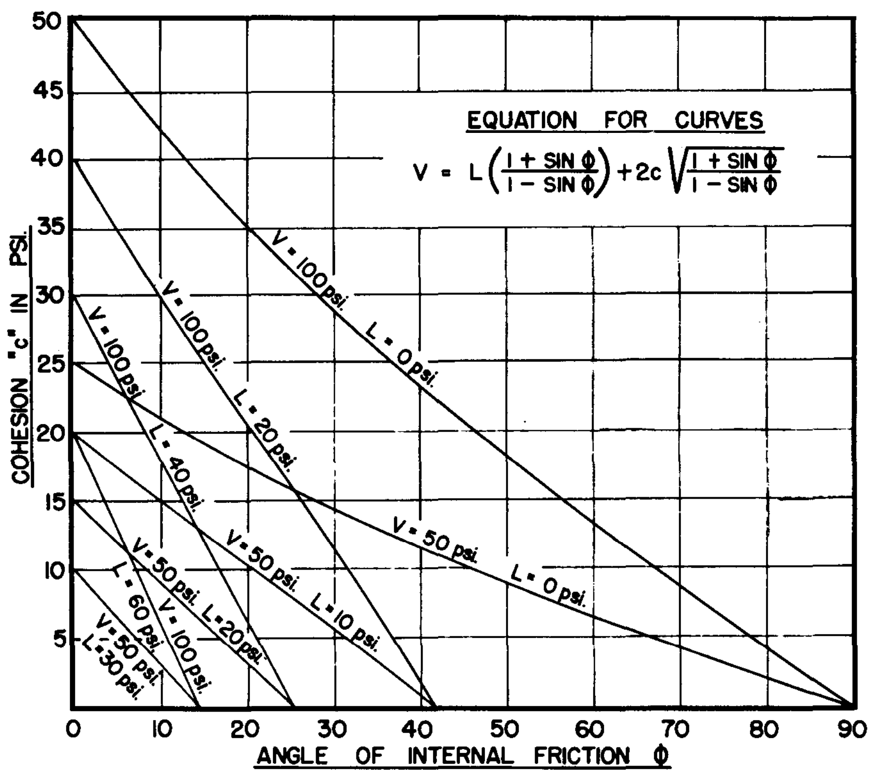



Figure 10. Relationships between c, $\phi$, L and V for Materials Having Positive Values for c and $\phi$ in Triaxial Compression

Figure 7, since for any given value of lateral support L, the most stable material is that which can support the largest vertical load V at failure. Equation (3) has the advantage over equation (2), that the maximum vertical load V that can be supported by any given material, is provided directly in terms L, c and $\emptyset$.

The stability diagram of Figure 10 is obtained, when equation (3) is plotted in terms of given values of vertical load V, for different magnitudes of lateral support L, and for various values of c and $\emptyset$.

Figure 11 illustrates the practical application of equation (3) and Figure 10 to the solution of a given stability problem. If a material having both cohesive and granular properties is to carry a vertical load V of 100 p.s.i., when the lateral support L is 30 p.s.i., what values of cohesion c and angle of internal friction $\emptyset$

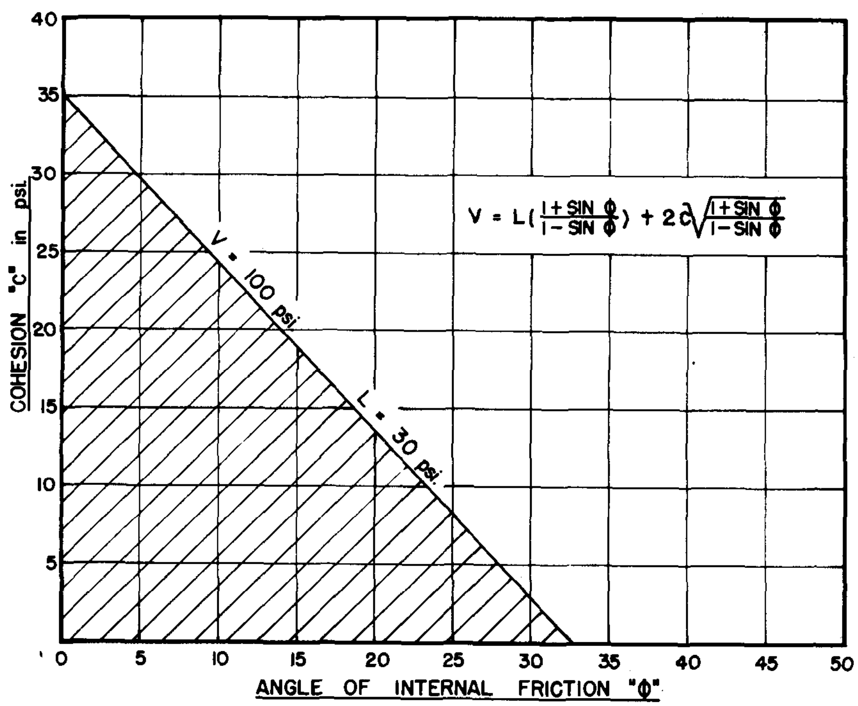



Figure 11. Stability Diagram in Terms of c, $\emptyset$, L and V for Materials Having Positive Values of c and $\emptyset$ in Triaxial Compression

are required? The graphical solution to this problem given in Figure 11 indicates that an infinite number of answers is possible. All materials possessing those combinations of c and Ø, which are on or to the right of the curve labelled V = 100 p.s.i., L = 30 p.s.i., would have the required stability. Materials with combinations of c and Ø that lie within the cross-hatched area to the left of this line would tend to be unstable, and therefore unsatisfactory insofar as this particular problem is concerned. It should be particularly noted that Figure 11 is intended to apply only to the material within an element carrying the particular stresses specified under equilibrium conditions. Figure 11 is not concerned with the nature of the material outside the stressed element, except that it must be placed in such a manner as to provide a lateral support L of 30 p.s.i.

## LATERAL SUPPORT PROVIDED BY PAVEMENT ADJACENT TO THE LOADED AREA

The general equations of stability (equations (2) and (3)), and the stability diagram of Figure 11 for a particular set of design requirements, V = 100 p.s.i. and L = 30 p.s.i., are not entirely satisfactory for the design of bituminous mixtures for pavements on highways and airports, for two reasons:

(a) Figure 11 demonstrates that they would permit the use of materials with very low and even zero cohesion c, since c becomes zero when Ø is about $32.5^{0}$. Experience has indicated that only those materials containing sufficient binder to provide an appreciable value for cohesion c, are capable of withstanding the particular types of stress to which the surface course is subjected by traffic.

(b) While the quantities c and Ø for any bituminous mixture can be measured by the triaxial test, no method for determining the value of the lateral support L that can be provided by the pavement surrounding the loaded area has been indicated. Unless values for lateral support L can be determined, stability equations (2) and (3) and the stability diagrams based upon these equations, e.g. Figures 9, 10 and 11, are of no practical value.

The minimum value of cohesion c required for bituminous mixtures for surface courses may not be particularly high. Some years ago, automobile speed record attempts were made on the sandy beach at Daytona Beach, Florida, at a certain time after the tide went out. The surface tension of the water retained in the sand over this critical period of time, was sufficient to provide the cohesion c (and stability) required for the test run. In addition, it is well known that the moisture films provided or maintained by the application of certain salts, e.g. calcium chloride, to the surfaces of stabilized gravel roads, prevent the damage to the surfaces of these roads that results when these moisture films are absent. Quantitative values of the cohesion c provided by the moisture films in these cases do not seem to be available, but they are probably not high.

A method based upon $V_i - L_i$ values, previously suggested by the writer[12] to establish minimum values of cohesion c for bituminous mixtures, may require c values that are higher than necessary. Probably the most reasonable method for obtaining the minimum values of c required for bituminous mixtures, would be to determine them experimentally by means of triaxial tests on samples from bituminous pavements that have performed differently in the field. For example, ravelling may be an indication of insufficient cohesion c in the paving mixture.

A method for determining the maximum value of the lateral support L provided by the portion of the pavement adjacent to the loaded area is illustrated in Figure 12. In Figure 12(a), the principal, shear, and normal stresses that are developed in a bituminous pavement under load are indicated, when the weight of the pavement material is neglected. As the stress caused by the vertical load V, develops the shearing resistance $s_c$ on the diagonal plane making an angle of $45 - \frac{\phi}{2}$ with the vertical in element (1), lateral pressure L is exerted in a horizontal direction on element (2) immediately adjacent to the loaded area. The maximum lateral pressure L that can be sustained by element (2), is determined by its shearing resistance $s_c$ acting along the diagonal plane making an angle of $45 - \frac{\phi}{2}$ with the horizontal. Figure 12(b) and (c) illustrate the principal, shear, and normal stresses acting on the isolated elements (1) and (2) respectively, neglecting their weight. Figure 12(d) is a Mohr diagram representing the stresses acting on element (2), for a bituminous paving mixture having the values of c and $\phi$ that result in the Mohr envelope indicated. The values of c and $\phi$ are determined directly from a triaxial test on the mixture. L is the major principal stress acting on element (2), and the minor principal stress is zero, if the weight of the element, and other factors, are neglected. The Mohr circle for these principal stresses is shown in Figure 12(d). This Mohr circle indicates that the maximum amount of lateral support L

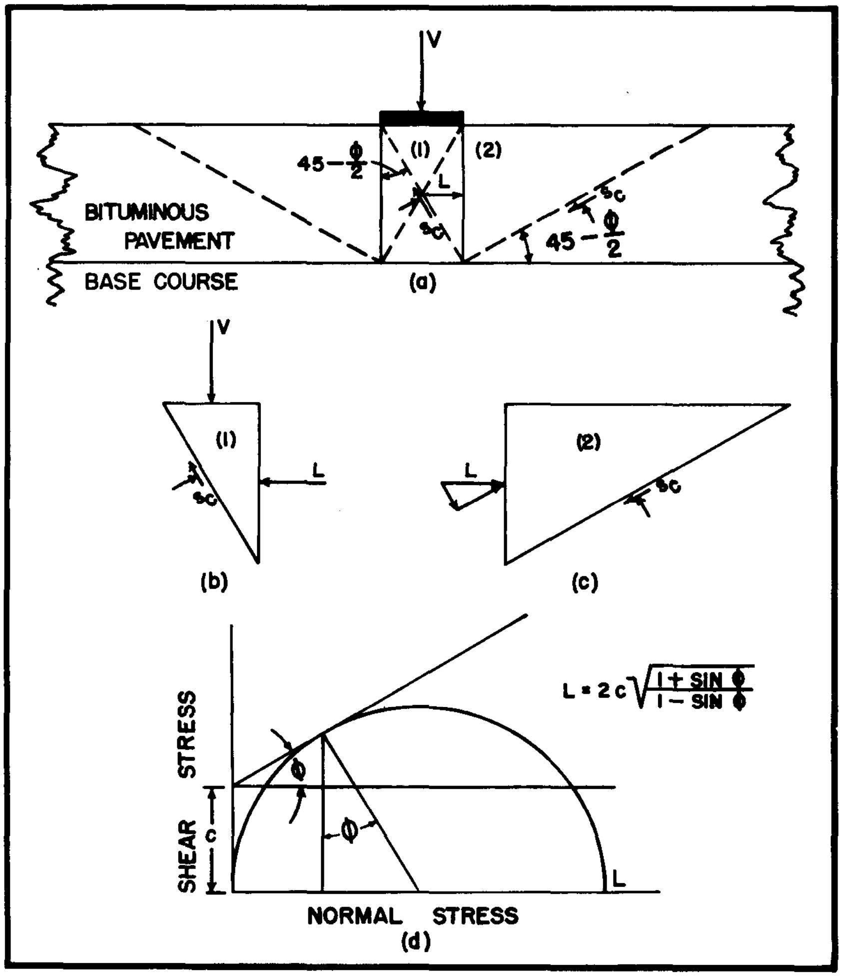



Figure 12. Illustrating that the Lateral Support L Provided by the Portion of a Bituminous Pavement Surrounding the Loaded Area is Given by $L = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}$

that can be developed by the portion of the pavement adjacent to the loaded area is equal to the unconfined compressive strength of the pavement mixture. From the geometry and trigonometry of the Mohr diagram illustrated in Figure 12(d), it is apparent that this value of lateral support L is given by

$$L = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \tag{4}$$

Substituting this value for lateral support L in equation (3), and simplifying, gives

$$V = \frac{4c}{1 - \sin \phi} \sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \tag{5}$$

Equation (5) is the equation of stability for a bituminous paving mixture, when it is assumed that the maximum lateral support L provided by the pavement adjacent to the loaded area is equal to the unconfined compressive strength of the paving mixture. This is illustrated in Figure 13.

Figure 14 is the stability diagram that results when equation (5) is plotted in terms of different values of vertical load V, and various magnitudes of c and $\phi$. If the vertical load V to be carried is 100 p.s.i. for example, Figure 14 indicates that only those bituminous mixtures having corresponding values of c and $\phi$ lying

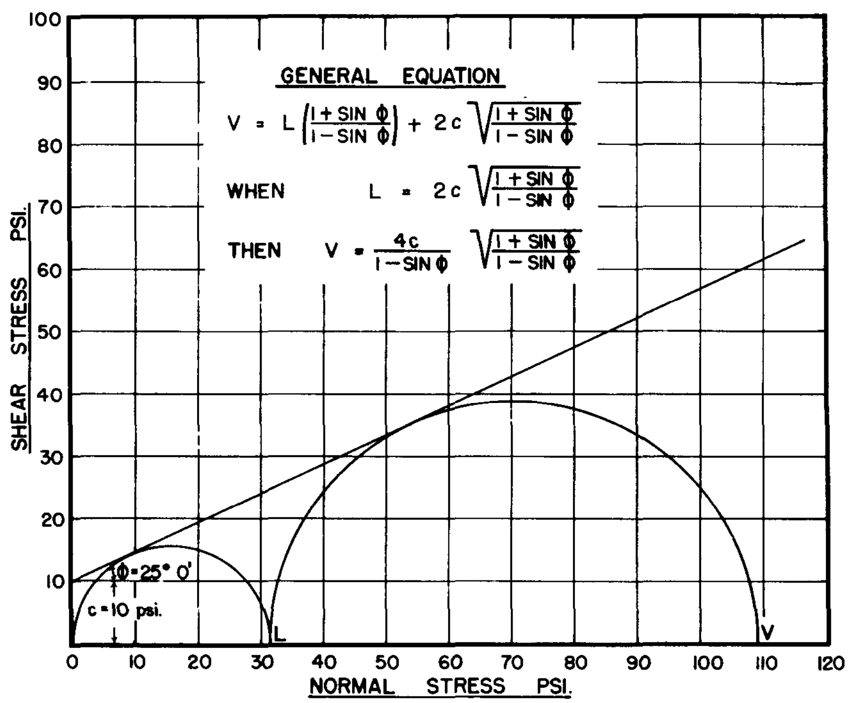



Figure 13. Diagram Illustrating Maximum Vertical Load V That Can Be Carried by a Bituminous Pavement When Lateral Support L is Equal to the Unconfined Compressive Strength of the Material

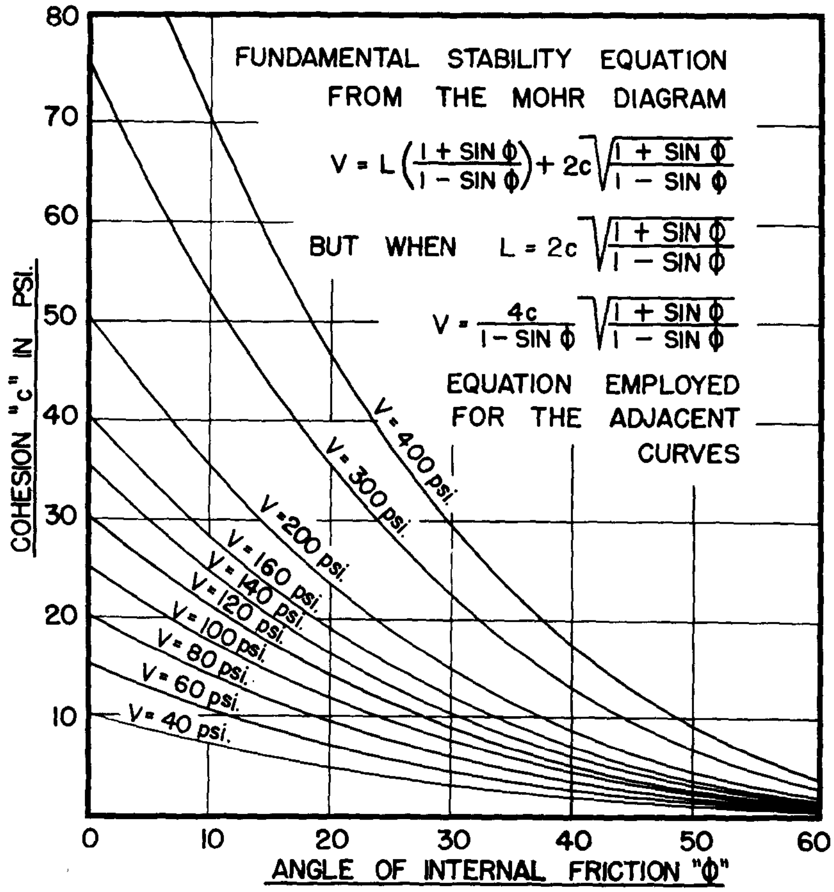



Figure 14. Design Chart for Bituminous Mixtures Based on Triaxial Test and Values of Lateral Support

$$L = 2c\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}$$

on or to the right of the curve labelled V = 100 p.s.i., will provide bituminous pavements with sufficient stability to carry this load.

It should be observed that equation (5), and Figure 14, provide an answer to the two main criticisms of equations (2) and (3) and Figures 9, 10, and 11, as a basis for the design of bituminous mixtures, that were previously mentioned. The curves in Figure 14, unlike those of Figures 9, 10, and 11, indicate that a certain amount of cohesion c is automatically provided for by the curve for each value of V throughout the diagram. In addition, equation (5) implies that the amount of lateral support L provided by the pavement adjacent to the loaded area, is equal to the unconfined compressive strength of the material.

While it was developed in connection with Figure 12 that the amount of lateral support L provided by the pavement adjacent to the loaded area is equal to the unconfined compressive strength of the paving mixture, the weight of element (2) was neglected. This development was also simplified by neglecting certain other factors that must now be considered, because they indicate that the actual amount of lateral support L provided by the pavement surrounding the loaded area may be appreciably greater than its unconfined compressive strength.

If the weight of element (2) in Figure 12(a) were taken into account, the minor principal stress acting on this element would not be zero as shown in Figure 12(d), but would have some positive value depending upon the density of the pavement. Reference to Figure 12(d) indicates that this would provide a value of lateral support L greater than the unconfined compressive strength, since the Mohr circle corresponding to a minor principal stress greater than zero would be to the right of that shown in Figure 12(d).

Figure 12(a) is based upon a strip loading and assumes that the stresses applied by a tire have a greater tendency to squeeze a bituminous pavement from under the wheel in a transverse direction, than longitudinally towards the front or behind the tire. Figure 15, drawn from data obtained by Teller and Buchanan[15], illustrates the basis for this assumption. Towards the front or rear of the contact area of a tire resting on a pavement, the pressure decreases from its full average value to zero over a much longer section of the area of contact, than is the case in the transverse direction. For bus and truck tires equipped with nearly flat treads, the pressure on the contact area probably decreases from its full average value to zero over a very narrow width in a transverse direction. Even with the more rounded treads usually employed for aeroplane tires, the data obtained by Teller and Buchanan[15] indicate that the pressure on the contact area decreases from its full average value to zero over a considerably greater width in a longitudinal than in a transverse direction, and this is supported by Porter's data for a very large aeroplane tire[16]. Consequently, for this reason, and for others that will be developed later in connection with Figures 16 and 39, a bituminous pavement tends to be less stable under the stresses applied by a tire which act in a transverse than in a longitudinal direction, and the former, presents the more critical conditions of stability to be considered

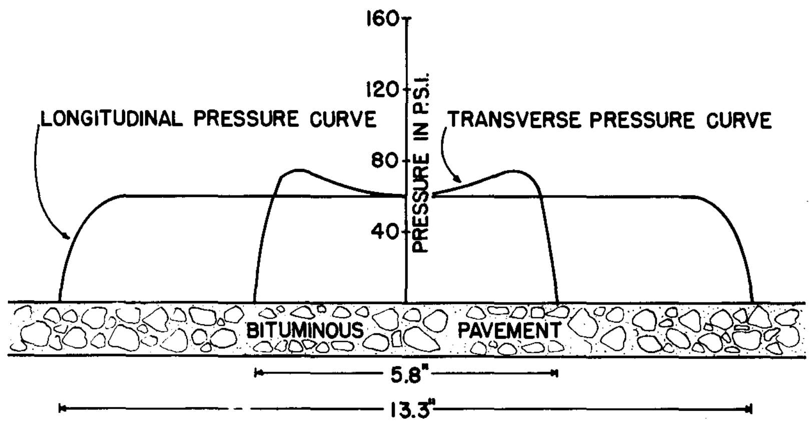



Figure 15. Demonstrating that Tire Pressure Near the Edge of the Contact Area Decreases From its Maximum Value to Zero More Rapidly in the Transverse Than in the Longitudinal Direction.

in the design of bituminous mixtures, at least for stationary loads, or for loads moving at a uniform rate of speed. That is, a stationary loaded tire resting on a bituminous pavement, or one moving at uniform speed, has a greater tendency to squeeze out the pavement in a transverse direction than towards the front or rear of the tire.

Strip loading assumes a relatively narrow loaded area of indefinite length, whereas the length of a tire contact area is rather short. If the contact area is over-loaded, there is a tendency for a whole wedge of the adjacent pavement, ABCDEF, Figure 16(a), to be forced out of the pavement. Consequently, in addition to the resistance due to shear along the diagonal plane ABCD just outside of the loaded area, which provides lateral support L under conditions of strip loading, shearing resistance along the triangular vertical end areas of the wedge, AED and BFC, is also developed. The shearing resistance along the vertical triangular end areas AGD and BHC should also be considered. Depending upon the length of the tire contact area and thickness of pavement, the shearing resistance of these triangular end sections might vary from a small percentage of the shearing resistance provided by the diagonal plane ABCD of Figure 16(a) for a large aeroplane tire, to a large percentage for a truck tire. That is, the shearing resistance provided by the vertical end sections AGD and BHC,

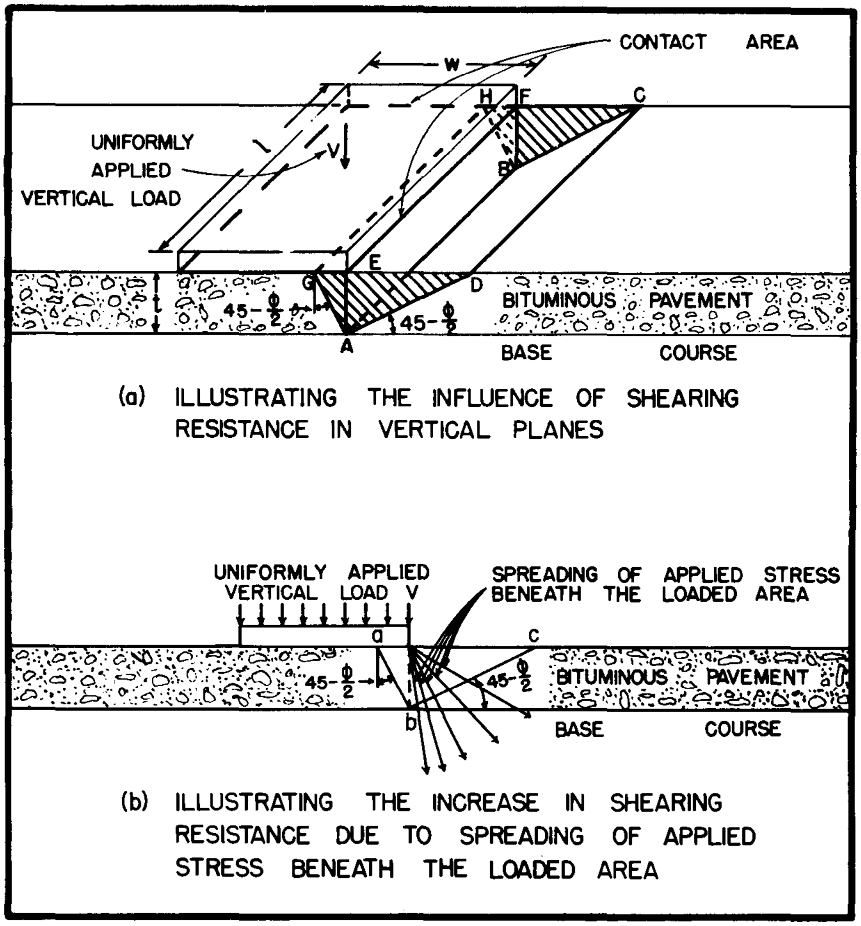



Figure 16. Illustrating Factors That Tend to Increase the Lateral Support L Above the Unconfined Compressive Strength of the Pavement

Figure 16(a), may increase the lateral support L provided by the pavement surrounding the loaded area by a considerable, but variable, percentage of the unconfined compressive strength.

It is apparent from Figure 16(a) that the ratio of the shearing area on the vertical triangular end sections AGD and BHC, versus that on the diagonal plane ABCD, would be higher in the direction of the longitudinal rather than the transverse axis of a tire's contact area, since the contact area is narrower in the longitudinal than in the transverse direction. That is, it would provide greater resistance to the squeezing out of a bituminous pavement from under a loaded tire toward the front or rear of the tire, than in the transverse direction.

In an actual pavement, the wedge of material under stress just outside the loaded area does not necessarily have the regularly defined shape illustrated by ABCDEF in Figure 16(a), since the contact area of a tire on a pavement is elliptical for aeroplane tires rather than rectangular as shown, although Paxson[17] has demonstrated that the contact areas for heavily loaded truck tires tend to be rectangular. Nevertheless, the important point to be considered in connection with Figure 16(a) is not the actual shape of the wedge of material under stress, but that shearing resistance is developed along the vertical end areas of the wedge, or their equivalent, and thereby increases the lateral support L available within the material surrounding the loaded area, beyond its unconfined compressive strength.

It has been known for many years that a load applied to the surface of a granular mass is spread out over a much wider area on any horizontal plane below the loaded area. This is demonstrated in Figure 16(b). It is quite apparent that because of the spreading of the load with depth, the diagonal shear plane bc is subjected to a considerably greater normal stress, than would be the case if this spreading out of the load did not occur. Housel[5] has observed this effect in connection with his investigations of the stability of granular materials. Because of this greater normal stress, the shearing resistance along the diagonal shear plane bc is larger than would be the case for an unconfined compression test. Consequently, because of the spreading of the applied load with depth, the lateral support L provided by the material surrounding the loaded area is greater than its unconfined compressive strength.

To summarize, therefore, the lateral support L provided by the pavement adjacent to the loaded area is greater than the unconfined compressive strength of the pavement because

(a) the weight of the pavement material increases the normal stress on the plane of failure,
(b) the area of the vertical and diagonal planes over which shearing resistance is developed is greater than would be the case if this resistance were limited to the unconfined compressive strength for strip loading,
(c) the outward spreading of load beneath the loaded area provides a normal stress on the plane of failure which is greater than that which would occur for unconfined compression.

These additional sources of lateral support L, provided by the pavement adjacent to the loaded area, can be taken into account by multiplying the unconfined compressive strength by a factor K. That is,

$$L = (K)\left(2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\right) \tag{6}$$

When the value of the lateral support L given by equation (6) is substituted in equation (3), we obtain on simplification

$$V = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\left[\frac{K(1 + \sin \phi) + (1 - \sin \phi)}{1 - \sin \phi}\right] \tag{7}$$

Figure 17 illustrates equation (7) in graphical form, and demonstrates the influence of different values of the factor K, varying

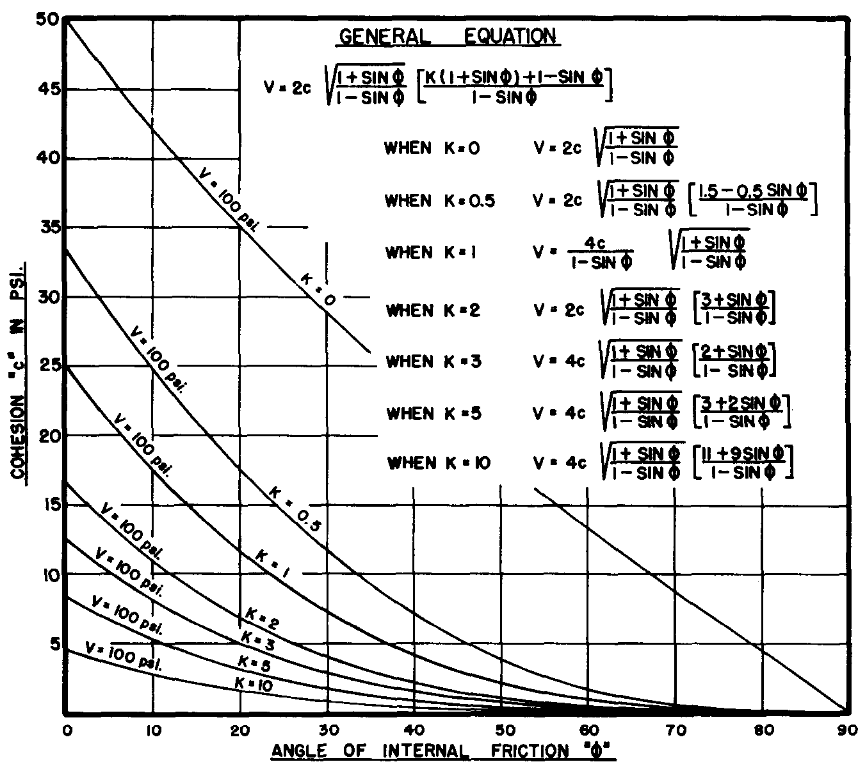



Figure 17. Diagram Illustrating the Influence of Different Values of K on the Design of Bituminous Mixtures

from 0 to 10, on the design of bituminous mixtures. When K = 1, equation (7) reduces to equation (5), so that the stability curve for V = 100 p.s.i., when K = 1 in Figure 17, is identical with the curve for V = 100 p.s.i. in Figure 14.

Stability curves in Figure 17 for V = 100 p.s.i., when K = 0 and when K = 1/2, apply to the condition where the wheel load is to be applied at or near the unsupported edge of a bituminous pavement. For a wheel load immediately at the unsupported edge of a pavement, the value of K to be employed would approach zero, since the amount of lateral support L provided under these conditions would also approach zero.

It will be observed in Figure 17 that each stability curve automatically specifies a minimum value of cohesion c for all values of internal friction $\phi$ less than 90°. Whether or not the values of cohesion c indicated by these curves would be adequate in all cases can only be determined from observations of the field performance of bituminous mixtures with known values of c and $\phi$. Practical experience might indicate the necessity for arbitrarily specifying some minimum value of cohesion c for all bituminous mixtures, for example 5 p.s.i. It seems more likely, however, that any arbitrarily specified value of cohesion c should vary with angle $\phi$ and with the applied vertical load V, in some manner to be determined experimentally on the basis of field performance.

The sources of lateral support L that are taken into account in equation (6) have reference to pavement stability in a transverse direction under a loaded area. For reasons previously outlined, pavement stability is considered to be more critical in a transverse than in a longitudinal direction under a stationary tire load, or under a wheel load moving at uniform speed. The sources of lateral support L in a longitudinal direction that are provided by the pavement adjacent to the loaded area, can be included by multiplying the unconfined compressive strength by the factor J, giving

$$L = (J)\left(2c\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}\right) \tag{8}$$

Since pavement stability appears to be more critical in a transverse than in a longitudinal direction, at least for stationary wheel loads, and for those moving at uniform speed, it seems reasonable to assume that the factor J in equation (8) is larger than K in equation (7). Thus, the latter equation represents a smaller and, therefore, more critical degree of lateral support L than the former.

## VISCOUS RESISTANCE

Cohesion c and angle of internal friction Ø are the two fundamental properties of bituminous mixtures that must be very carefully measured by the triaxial test. As a matter of fact, it is the ultimate objective of a method of design based on the triaxial test to determine the minimum corresponding values of c and Ø that will provide a stable bituminous pavement for the particular conditions associated with any given project.

The bituminous binder employed for bituminous mixtures is a very viscous material. When a bituminous paving mixture is deformed, this highly viscous binder provides the mixture with a "viscous resistance" that is proportional to the rate of deformation. The magnitude of this "viscous resistance" depends, therefore, upon the rate of strain or rate of loading employed when making the triaxial test. The "viscous resistance" of bituminous mixtures becomes important in pavement design when considering the stability of bituminous pavements subjected to rapidly moving loads.

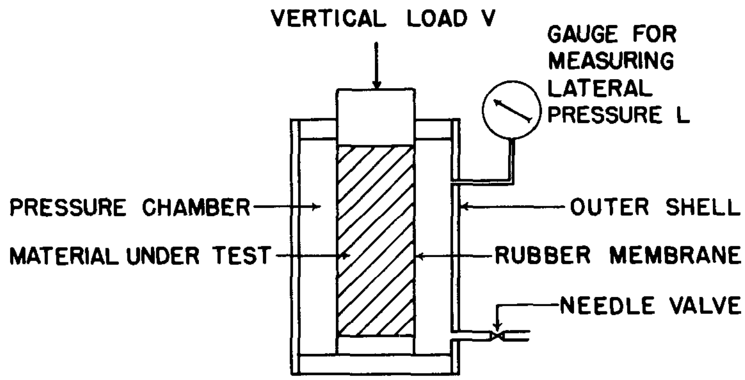
Consequently, the strength or stability of bituminous mixtures depends upon the magnitude of the three fundamental sources of stability that they possess,

(a) cohesion c
(b) angle of internal friction Ø
(c) viscous resistance

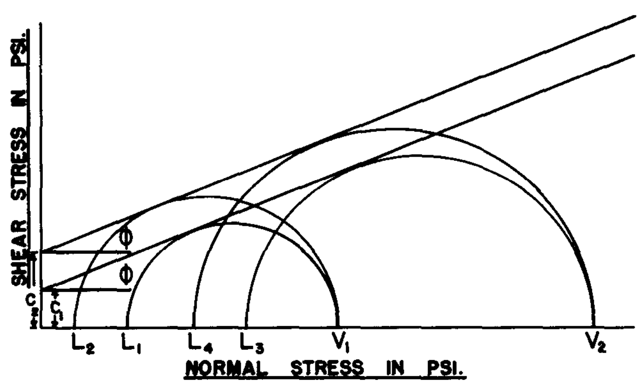
Values for cohesion c and angle of internal friction Ø for any given bituminous mixture are obtained by plotting triaxial data in the form of a Mohr diagram, but how is this "viscous resistance" factor to be evaluated in quantitative terms? A satisfactory approach to this problem seems to be indicated by the work of Nijboer[2] and is briefly outlined here.

For investigating the stability of bituminous mixtures, Nijboer employs the "cell" triaxial test, Figure 18(a), devised by Buisman at Delft, Holland, some years ago. An outstanding advantage of the "cell" triaxial test is that a complete Mohr diagram can be obtained with a single test specimen, instead of the three or four specimens required by the more standard type of triaxial equipment employed in North America, illustrated in Figure 2. The cell triaxial test is described, and its use is illustrated, by reference to Figure 18(a) and (b).

For the cell triaxial test, the prepared specimen is placed in the rubber membrane in the apparatus shown in Figure 18(a). Water or other suitable liquid is pumped into the annular space




(a) DIAGRAM OF CELL TRIAXIAL APPARATUS




(b) MOHR DIAGRAM ILLUSTRATING INFLUENCE OF VISCOUS RESISTANCE ON STABILITY OF BITUMINOUS MIXTURES

Figure 18. The Measurement of Viscous Resistance and its Influence on the Stability of Bituminous Mixtures

between the specimen and the outer cylindrical wall. When this space is filled, the valve is closed so that no liquid can escape. Specimen and liquid should be maintained at the desired testing temperature considered to be critical for the region in which the pavement is to be built.

A constant vertical load $V_1$ is applied to the test specimen. The specimen deforms rapidly at first under this constant load and builds up lateral pressure L in the surrounding liquid, which cannot escape. Finally the rate of deformation slows to zero (no vertical movement under the constant vertical load $V_1$), and the lateral pressure reaches its maximum value $L_1$, which is read off the pressure gauge. At the time this reading is made, the rate of deformation of the specimen is zero, so no viscous resistance is developed within the specimen. The values for $V_1$ and $L_1$ can be marked on the abscissa of the Mohr diagram, Figure 18(b) and the resulting Mohr circle drawn.

With the constant vertical load $V_1$ still being maintained, the needle valve is opened slightly to drain off liquid from the space surrounding the specimen at a rate that causes it to be deformed in a vertical direction at some desired constant rate of strain. This constant rate of vertical deformation of the specimen brings its viscous resistance into play, and the constant vertical load $V_1$ is now resisted partly by the lateral pressure L of the surrounding fluid, and partly by the developed viscous resistance of the material. Therefore, the lateral pressure drops below its previous value $L_1$ and, when it has become constant under these conditions, its value $L_2$ is read from the pressure gauge. The value for $L_2$ is marked on the Mohr diagram, Figure 18(b), and the Mohr circle for $V_1$ and $L_2$ is described as shown.

The needle valve is closed and the same procedure is repeated for a new constant vertical load $V_2$. The Mohr circles for $V_2$ and $L_3$, and for $V_2$ and $L_4$, are drawn on the Mohr diagram, Figure 18(b).

This procedure should be repeated for still higher values of constant vertical load V to obtain additional Mohr circles, in order that the Mohr envelopes may be established with greater accuracy. Several different rates of vertical deformation may also be employed, to investigate the behaviour of any proposed bituminous paving mixture over a wide range of conditions.

Mohr envelopes are drawn tangent to the Mohr circles $V_1L_1$ and $V_2L_3$, and to $V_1L_2$ and $V_2L_4$, Figure 18(b). It will be observed that the angle of internal friction $\phi$ is the same for both Mohr envelopes. It has been shown this way in Figure 18(b), because Nijboer[2] reports a theoretical study (supported by some test data), which indicates that the angle of internal friction $\phi$ should be independent of the rate of deformation of the bituminous mixture, as long as the air voids are not below the critical minimum, which is usually 2 to 3 per cent. With respect to cohesion c on the other hand, it will be noted that cohesion $c_2$ for the Mohr envelope for Mohr circles $V_1L_2$ and $V_2L_4$ is much greater than cohesion $c_1$ for the Mohr envelope for Mohr circles $V_1L_1$ and $V_2L_3$, Fig. 18(b).

Consequently, Figure 18(b) indicates that the viscous resistance developed by deforming bituminous paving mixtures at any constant rate of strain is represented by an increase in the value of cohesion c obtained for the mixture.

Mr. John Walter, Assistant Highway Engineer, Department of Highways of Ontario, very kindly provided the triaxial data that form the basis for Figure 19. It illustrates the influence on the

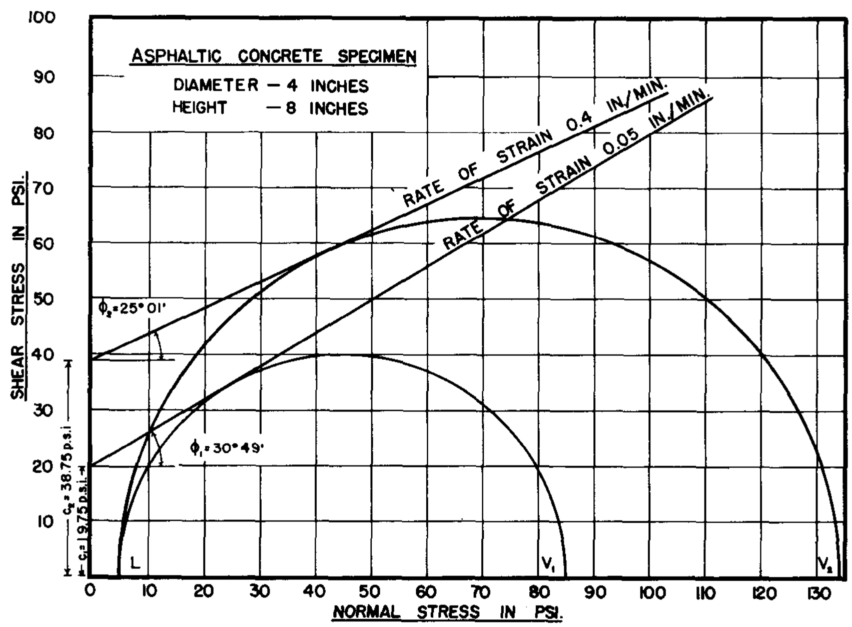



Figure 19. Influence of Rate of Strain on the Stability of Bituminous Mixtures

Mohr envelope, when the rate of strain employed for a triaxial test on a bituminous concrete paving mixture was increased from 0.05 to 0.4 inch per minute, that is, one crosshead of the testing machine moved at rates of 0.05 and 0.4 inch per minute, respectively, with reference to the other. The test specimens used were identical in every respect, and were 8-in. high. by 4-in. in diameter. The open triaxial apparatus illustrated in Figure 2 was employed.

The Mohr envelopes of Figure 19 demonstrate that the value of cohesion c obtained for this bituminous mixture was practically doubled from 19.75 p.s.i. to 38.75 p.s.i., when the rate of strain was increased from 0.05 to 0.4 inch per minute. The angle of internal friction Ø is shown to have decreased several degrees for the higher rate of strain. This is not in harmony with Nijboer's conclusions from his theoretical study, that the magnitude of the angle of internal friction Ø is independent of the rate of strain, Figure 18(b). However, the air voids in the specimens with which Mr. Walter's laboratory was working to provide the data for Figure 19 averaged 2.87 per cent. This is within the critical range of 2 to 3 per cent of air voids wherein Nijboer has indicated that the value of Ø may not be independent of the rate of strain employed to test the bituminous mixture. More laboratory work is required to determine whether Nijboer's theoretical deductions hold in all cases, or whether the difference in the angle of internal friction Ø indicated in Figure 19 may be representative of the behavior of some bituminous mixtures, even at contents of air voids appreciably greater than 3 per cent.

Goetz and Chen[4] have obtained some data that tend to indicate that the magnitude of the angle of internal friction Ø is not affected by the rate of strain, but that the value of cohesion c increased steadily as the rate of strain was increased from 0.05 to 2.0 inches per minute.

In connection with Figure 19, it is of more than usual interest to observe that the rate of strain employed for the Hubbard-Field stability test for an asphaltic concrete briquette 3 inches in thickness is 2.4 inches per minute[18]. For the specimens 8 inches high on which Figure 19 is based, the rate of strain corresponding to the Hubbard-Field procedure would be 6.4 inches per minute. The Marshall test employs a rate of strain of 2 inches per minute for a specimen 4 inches high[19]. This corresponds to a rate of strain of 4 inches per minute for a specimen having a height of 8 inches. It is apparent that if the same bituminous mixture employed for Figure 19 were tested triaxially at rates of strain of 6.4 and 4 inches per minute, corresponding to Hubbard-Field and Marshall test requirements, respectively, the resulting Mohr envelopes and values for cohesion c might be above the top of the diagram of Figure 19. Consequently, there seems to be considerable justification for the criticism frequently made of both Hubbard-Field and Marshall tests, that the stability values they provide are influenced very largely by the cohesion c of bituminous mixtures.

For the Hveem stabilometer[6], the rate of strain employed is 0.05 inch per minute for a specimen about 2 5/16 inches high. This corresponds to a rate of strain of slightly less than 0.2 inch per minute for a specimen 8 inches tall. For this rate of strain, the resulting Mohr envelope would be between those shown in Figure 19.

Smith[3] has recommended that bituminous mixtures be tested at zero rate of strain. This would give a Mohr envelope somewhat below that for the strain rate of 0.05 inch per minute illustrated in Figure 19.

The above examples indicate that in terms of test specimens 8 inches high, stability tests in common use at the present time specify rates of strain varying from 0 to 6.4 inches per minute. It is apparent from Figure 18(b) and 19 that, insofar as the triaxial test is concerned, little correlation of stability values measured by different laboratories can be expected until rates of strain are standardized.

Stability equations (5) and (7) show that cohesion c is a multiplier for the balance of the expression on the right hand side of each equation. Therefore, any triaxial testing procedure that specifies an unduly high rate of strain will provide exaggerated values of cohesion c, which, when substituted in stability equations (5) and (7), may indicate stability values that are far greater than the paving mixture is actually able to develop under field conditions. For example, suppose that two samples of a given bituminous mixture are both reported to have an angle of internal friction $\phi = 24^{\circ}$, when tested in two independent laboratories. In one laboratory the rate of strain employed is 0.05 inch per minute and the value found for cohesion c = 10 p.s.i., while the other laboratory employed a rate of strain of 0.4 inch per minute and found cohesion c = 20 p.s.i. Figure 14 indicates that the stability rating for this bituminous mixture would be 100 p.s.i. on the basis of data reported by one laboratory, and 200 p.s.i. according to the data supplied by the other, merely because they did not both employ the same rate of strain. Consequently, Figures 18(b) and 19 emphasize the necessity for selecting rates of strain, when testing bituminous mixtures in the laboratory, that correspond to the rates of deformation to which they will be subjected under traffic in the field.

For a truck wheel carrying a load of 5,000 pounds and a tire pressure of 70 p.s.i., any given point on the pavement over which this tire passes is subjected to a load of 70 p.s.i. for about 0.6 second if the truck is travelling at 1 mile per hour, and for about 0.01 second if it is travelling at 60 miles per hour. Even with a large aeroplane tire inflated to 100 p.s.i. and carrying a load of 100,000 pounds, a given point on a pavement over which it passes is subjected to this pressure for only about 0.03 second when travelling at 100 miles per hour, and for about 2 seconds when travelling at 1 mile per hour.

It is clear, therefore, that insofar as moving vehicles are concerned, bituminous pavements are subjected to loads of very short duration, and the viscous resistance developed by the bituminous mixture must be quite high. This would be equivalent to conducting a laboratory stability test at a high rate of strain. It might seem, therefore, that a reasonably high rate of strain would be justified for the stability testing of bituminous mixtures to be employed where moving traffic is expected, such as airport runways, and for highways, apart from bus stops and traffic lights. However, many pavements on airport runways, on highways, and on city streets, are subject to braking and acceleration stresses. The influence of these on pavement design is considered in a later section of this paper.

Pavements for stationary or extremely slow moving traffic should be designed on the basis of laboratory stability tests performed at a very low rate of strain. Nijboer[2] has observed that a pneumatic tire resting on a bituminous pavement 2 inches thick that was giving satisfactory service, settled about 1 mm. into the pavement in 30 minutes. He, therefore, recommends that the rate of strain employed for triaxial tests should be about 0.005 inch per minute for specimens 8 inches high. This rate of strain would result in a Mohr envelope somewhat below the lower Mohr rupture line in Figure 19. The rate of strain recommended by Nijboer for pavement design for stationary loads may not be unreasonable. Nevertheless, it is clear that the rate of strain to be employed for laboratory triaxial tests requires further careful consideration by everyone interested in this topic.

Nijboer[2] indicates that the limits of accuracy when testing successive samples of a given paving mixture with the cell triaxial apparatus are 30' for the angle of internal friction $\phi$, and 10 per cent, or a minimum of 1.4 pounds per square inch, whichever is greater, for cohesion c.

## INFLUENCE OF FRICTIONAL RESISTANCE BETWEEN PAVEMENT AND TIRE AND BETWEEN PAVEMENT AND BASE

In a previous section, the influence on pavement stability of the lateral support L provided by the pavement adjacent to the loaded area was considered. However, there are frequent examples where even at the complete exposed and unsupported edge of a bituminous pavement, no indications of instability have developed after years of traffic. These unsupported pavement edges are stable under traffic, either because of the high compressive strength of the paving mixture, or because bituminous pavements can develop additional resistance to lateral flow, quite apart from the lateral support normally provided by adjacent pavement material, or both. Figure 20 indicates that there is a further source of resistance to the lateral movement of a pavement under a loaded tire, that must be considered.

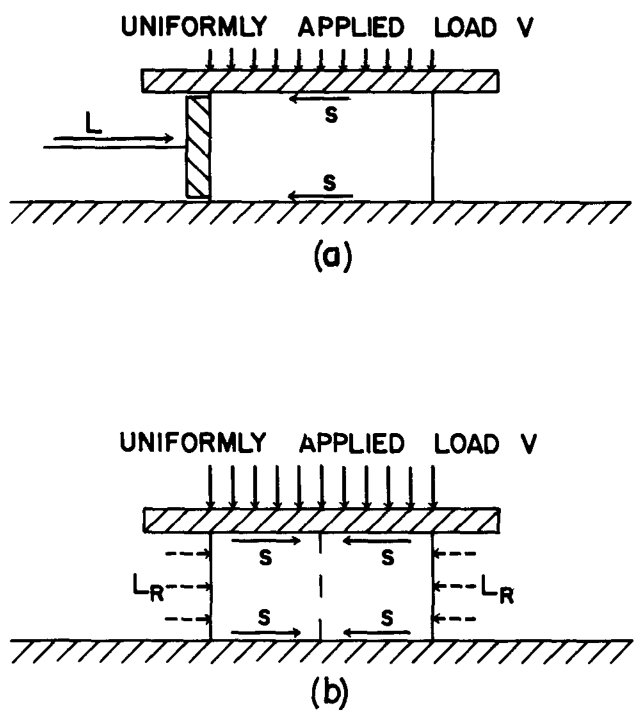



Figure 20. Diagram Illustrating That Friction between Tire and Pavement and between Pavement and Base is Equivalent to Additional Lateral Support for the Section of Pavement Under a Loaded Area

Figure 20(a) illustrates the resistances developed when a horizontal force L is applied to an isolated section of bituminous pavement held between two rough flat surfaces carrying a vertical load. It is apparent that the horizontal pressure L applied as shown will develop frictional resistance "s" between the pavement and each of the two rough surfaces. That is, frictional resistance can be developed between the pavement and the two rough surfaces equivalent to a horizontal pressure L.

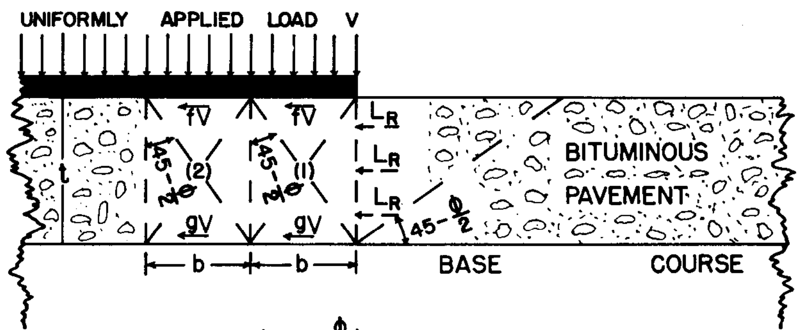
In Figure 20(b), on the other hand, the section of bituminous pavement is subjected to sufficient vertical load V to cause it to flow laterally. This is equivalent to the movement of an overloaded bituminous pavement beneath a tire. Figure 20(b) demonstrates that as the paving mixture is being squeezed out, its lateral movement is opposed by the frictional resistance "s" developed between pavement and tire and between pavement and base. It is apparent from both Figures 20(a) and (b) that this frictional resistance between pavement and tire and pavement and base is equivalent to a lateral support $L_R$.

If this frictional resistance is to be utilized for the design of bituminous mixtures, it must be evaluated quantitatively and taken into account in equations of design and when constructing charts of design curves. Figure 21 illustrates a method for evaluating this frictional resistance in terms of an equivalent lateral support $L_R$.

Krynine[20] refers to the work of Jurgenson[21], which shows that when a material is squeezed between two rough parallel plates, the shearing stress developed in the material is at a minimum on the plane midway between and parallel to the two plates. The maximum shearing stress occurs at the interfaces between the plates and the material. Therefore, in the case of a bituminous pavement squeezed between a tire and the base course, it seems reasonable to assume that the maximum shearing stress is developed at the interfaces between pavement and tire and between pavement and base.

It should be clear that the maximum frictional resistance that can be developed at either of these two interfaces cannot exceed the shearing resistance of the pavement itself. That is, the maximum frictional resistance that can be developed between the pavement and tire is the lesser of either the coefficient of friction "f" between pavement and tire multiplied by the vertical pressure V, that is, fV, or the shearing resistance of the bituminous mixture given by the Coulomb equation $s = c + V \tan \phi$, where V is the normal pressure. Similarly, the maximum frictional resistance that can be mobilized between the pavement and base is the lesser of either the coefficient of friction "g" between pavement and base multiplied by the normal pressure V, that is, gV, or the shearing resistance of the bituminous mixture given by the Coulomb equation $s = c + V \tan \phi$.

Due to the spreading of load with depth, the actual vertical pressure at the top of the base course may be somewhat less than the applied load V at the surface. This is a refinement that it may




$$b = t \, \text{TAN}(45 - \frac{\phi}{2})$$

$fV = P(c + V\text{TAN}\,\phi)$ WHERE $P \leq 1$

$gV = Q(c + V\text{TAN}\,\phi)$ WHERE $Q \leq 1$

FOR ELEMENT (1)

MAXIMUM LATERAL SUPPORT $L_R$ DUE TO FRICTIONAL RESISTANCE BETWEEN TIRE AND PAVEMENT AND BETWEEN PAVEMENT AND BASE COURSE IS GIVEN BY

$$L_R = \frac{P(c + V\text{TAN}\,\phi)b + Q(c + V\text{TAN}\,\phi)b}{t}$$

$$= (P+Q)(c + V\text{TAN}\,\phi)\left[\text{TAN}(45 - \frac{\phi}{2})\right]$$

FOR ELEMENT (2)

$$L_R = 2(P+Q)(c + V\text{TAN}\,\phi)\left[\text{TAN}(45 - \frac{\phi}{2})\right]$$

FOR ELEMENT (n)

$$L_R = n(P+Q)(c + V\text{TAN}\,\phi)\left[\text{TAN}(45 - \frac{\phi}{2})\right]$$

Figure 21. Diagram Illustrating the Magnitude of the Lateral Support $L_R$ Equivalent to the Frictional Resistance Developed between Tire and Pavement and between Pavement and Base, under the Loaded Area

be necessary to consider at a later date. For the present paper, however, since the usual bituminous pavement is seldom more than from two to four inches thick, the vertical pressure transmitted to the top of the base course is assumed to be equal to the applied pressure V at the surface of the pavement, as a reasonable approximation.

The limitation on the maximum value of fV that can be developed can be expressed by letting

$$\frac{fV}{c + V \tan \phi} = P \qquad \text{Where } P \leqq 1 \tag{9}$$

from which

$$fV = P(c + V \tan \phi) \tag{10}$$

Similarly, the limitation on gV can be expressed by letting

$$\frac{gV}{c + V \tan \phi} = Q \qquad \text{Where } Q \leqq 1 \tag{11}$$

from which

$$gV = Q(c + V \tan \phi) \tag{12}$$

Equations (9) and (11) state that the coefficient of friction between pavement and tire, f, and between pavement and base, g, multiplied by the normal stress V, cannot exceed the shearing resistance of the bituminous paving mixture, although the reverse could occur, both of which, of course, are true in actual practice. The terms P and Q express the values of the ratios of equations (9) and (11). It is apparent that the highest value that either P or Q can have individually is unity and the lowest value is zero. Therefore, the maximum value for $P + Q = 2$, and the minimum value for $P + Q = 0$.

For reasons that cannot be foreseen, or that have been overlooked, quantities sometimes develop larger values in actual practice than previous logical reasoning has indicated to be possible for them. Consequently, it might turn out later that either one or both of the factors P and Q are capable of developing larger values than unity under certain conditions. However, since there is no present reason for expecting this, it is assumed for the present paper that, as indicated by equations (9) and (11), neither of the quantities P and Q can have values greater than unity.

The values of the coefficient of friction f between tire and pavement have been measured by Moyer[22] and by Giles and Lee[23]. They report values of f up to 1.0 for stationary or slowly moving vehicles, although 0.8 is a more normal top value. Moyer's data indicate that the value of the coefficient of friction f drops appreciably as the speed of the vehicle increases. No data are available concerning the value of g, the coefficient of friction between pavement and base. For a rational method of design, values for f and g must either be determined or assumed for pavement design for each individual project.

In Figure 21 the pavement under the loaded area has been divided into several elements, numbered inward from the edge. It is assumed that the pavement material under the loaded area tends to fail along planes making an angle of $45 - \frac{\phi}{2}$ with the vertical.

If the thickness of the pavement is t inches, and if the width of each element under the loaded area is b inches, it is apparent that,

$$b = t \tan\left(45 - \frac{\phi}{2}\right) \tag{13}$$

from which it follows that the ratio

$$\frac{b}{t} = \tan\left(45 - \frac{\phi}{2}\right) \tag{14}$$

The other equations for determining the value of the lateral support $L_R$, that is equivalent to the frictional resistance between pavement and tire, and between pavement and base, are listed in Figure 21, for successive elements of the pavement numbered inward from the edge of the loaded area. The general equation for $L_R$ for the $n^{th}$ element from the edge is,

$$L_R = n\,(P + Q)\,(c + V \tan \phi)\left(\tan\left(45 - \frac{\phi}{2}\right)\right) \tag{15}$$

Consequently, the total lateral support L that can be mobilized for the stability of bituminous mixtures is given by the sum of the lateral support $L_S$ provided by the pavement adjacent to the loaded area, plus the lateral support $L_R$ equivalent to the frictional resistance between pavement and tire and between pavement and base, or

$$L = L_S + L_R \tag{16}$$

where $L_S$ is given by equations (6) or (8), and $L_R$ by equation (13).

When the expression for the total lateral support L given by equation (16) is substituted in equation (3), the result is given by,

$$V = 2cK\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) + 2c\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}} + n\,(P + Q)\,(c + V \tan\phi)\left(\tan\left(45 - \frac{\phi}{2}\right)\right)\left(\frac{1 + \sin\phi}{1 - \sin\phi}\right) \tag{17}$$

which on simplification becomes,

$$V = c\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}\left[\frac{n\,(P + Q) + 2\,(1 - \sin\phi) + 2K\,(1 + \sin\phi)}{1 - \{1 + n\,(P + Q)\}\sin\phi}\right] \tag{18}$$

Equation (18) may be written in terms of f + g, rather than P + Q,

$$V = 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\left[\frac{K(1 + \sin \phi) + 1 - \sin \phi}{1 - \sin \phi - n(f + g)\cos \phi}\right] \tag{19}$$

It is instructive to determine the stability of successive elements of a bituminous pavement inward from the edge of the loaded area. This is necessary if the least stable element under the loaded area is to be ascertained.

Equations (18) and (19) cannot be used for this purpose, because they apply only to the determination of the corresponding values of c and $\phi$ required to support some specified unit load V uniformly applied to the contact area, when values for n, K, P + Q and f + g are also given, e.g. Figure 28. For investigating the change in stability across the loaded area, on the other hand, a bituminous mixture with given values of c and $\phi$ must be considered, and the pavement stability developed at various points on the contact area may be quite different from the uniformly applied unit load. To determine the stability of successive elements of a bituminous pavement inward from the edge of the loaded area, therefore, it is necessary to rewrite equations (18) and (19) in somewhat different form, when they become,

$$V = 2cK\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) + 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} + n(P + Q)(c + V' \tan \phi)\left(\tan\left(45 - \frac{\phi}{2}\right)\right)\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) \tag{20}$$

and

$$V = 2cK\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) + 2c\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} + nV'(f + g)\left(\tan\left(45 - \frac{\phi}{2}\right)\right)\left(\frac{1 + \sin \phi}{1 - \sin \phi}\right) \tag{21}$$

where

V = the stability developed by the bituminous pavement at any point on the contact area

V' = the unit vertical load uniformly applied to the contact area

and the other symbols have the significance previously defined for them.

It should be noted that in equations (18) and (19) V = V', whereas this is not generally true for equations (20) and (21).

For specified values for c, $\phi$, K, P + Q, f + g, and V', it should be apparent that the value of V in equations (20) and (21) varies directly and linearly with the value of n, where n indicates the distance measured in unit elements, Figure 21, from the edge to the point on the contact area at which the stability value V is required. This is illustrated by the straight line stability curves in Figures 22, 23, and 24, for a large aeroplane tire, and Figure 25 for a truck tire.

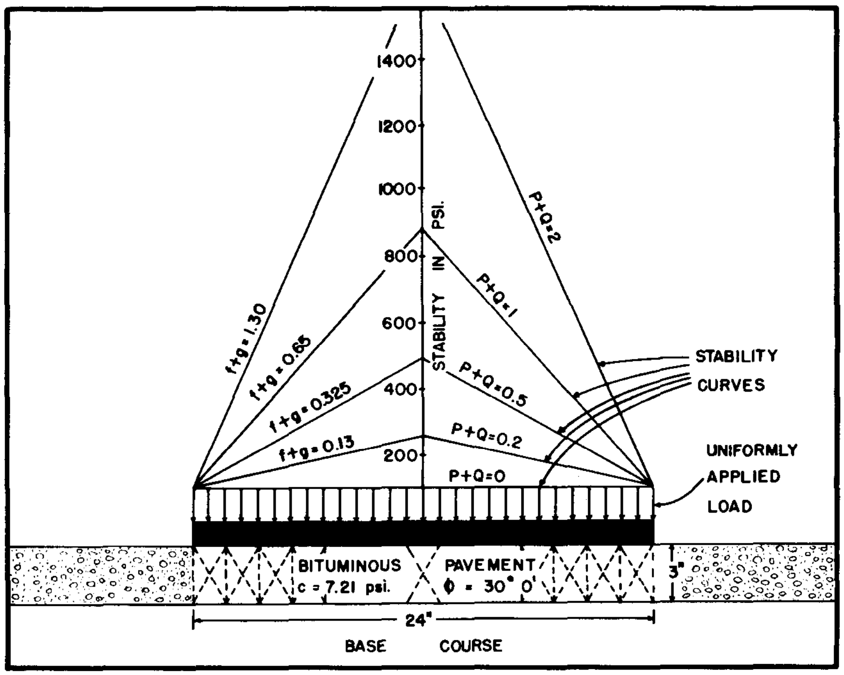



Figure 22. Relationships between Applied Load and Stability of Bituminous Pavements at Varying Distances from Edge Under the Loaded Area and for Different Degrees of Frictional Resistance Developed between Pavement and Tire and between Pavement and Base. (Pavement Stability Equal to Applied Load for Edge Conditions) Aeroplane Tire

Identical stability curves are drawn on both right hand and left hand sides in each of these four figures. However, the stability curves on the right hand side were calculated by means of equation (20), using the values of P + Q indicated, while equation (21) was employed for those on the left hand side, utilizing the corresponding f + g values. Therefore, Figures 22, 23, 24, and

25 illustrate the corresponding numerical values of P + Q and f + g that apply to each curve in any given set of stability curves, when all other factors are the same.

The values of c = 7.21 p.s.i. and $\phi$ = 30° enable the bituminous pavement in Figure 22, to just support an applied vertical load of 100 p.s.i. on an aeroplane tire contact area 24 inches wide, if the lateral support L is equal to the unconfined compressive strength of the pavement (K = 1 in equation (6)). Consequently, if no frictional resistance is developed between pavement and tire and between pavement and base (P + Q = 0 or f + g = 0), this pavement will just carry the applied vertical load V of 100 p.s.i. However, if this frictional resistance is only great enough to make P + Q = 0.2 (equations (9) and (11)), or f + g = 0.13, it is apparent from

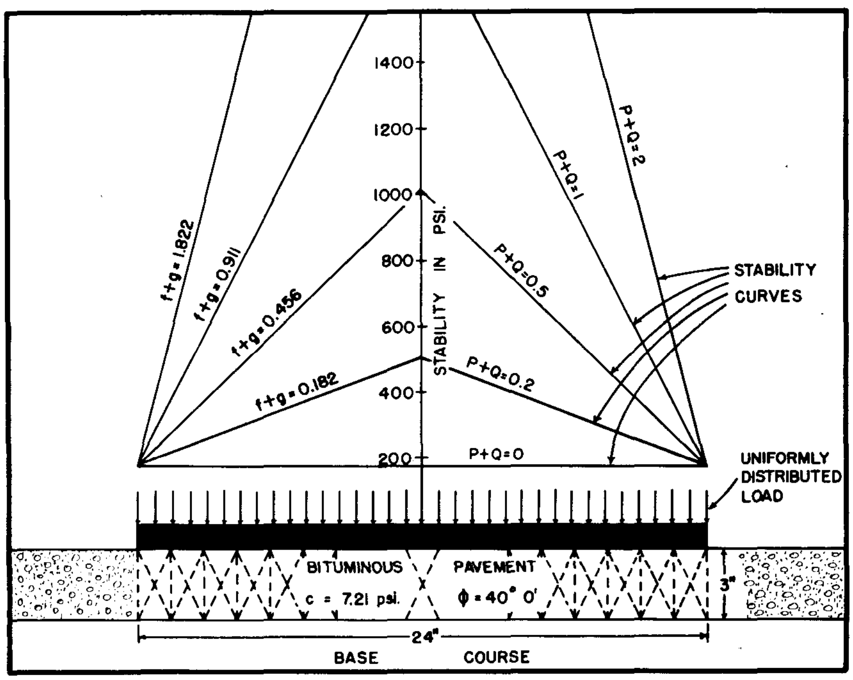



Figure 23. Relationships between Applied Load and Stability of Bituminous Pavements at Varying Distances from Edge Under the Loaded Area and for Different Degrees of Frictional Resistance Developed between Pavement and Tire and between Pavement and Base. (Pavement Stability Greater than Applied Load for Edge Conditions) Aeroplane Tire

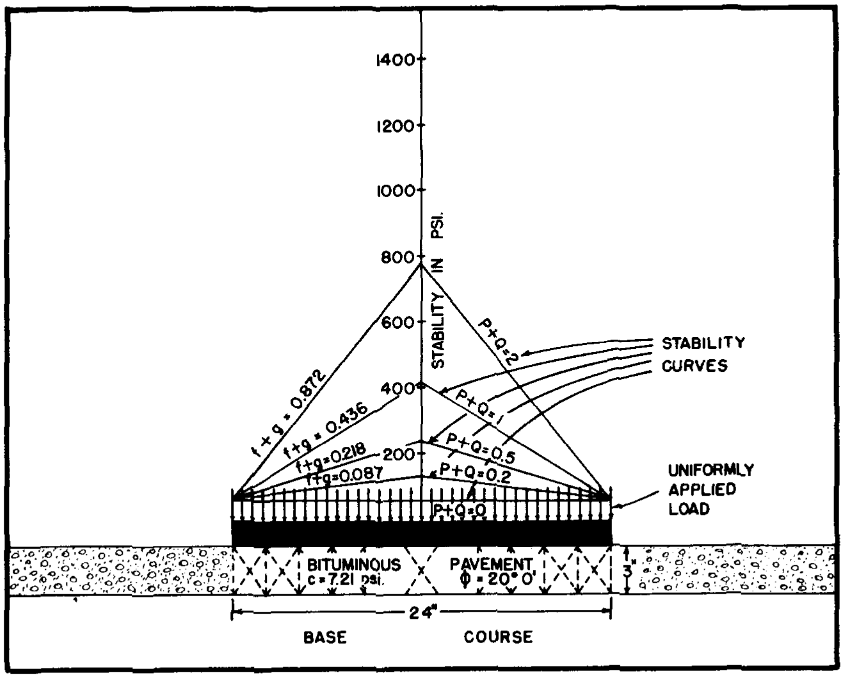



Figure 24. Relationships between Applied Load and Stability of Bituminous Pavements at Varying Distances from Edge Under the Loaded Area and for Different Degrees of Frictional Resistance Developed between Pavement and Tire and between Pavement and Base. (Pavement Stability Less Than Applied Load for Edge Conditions) Aeroplane Tire

the stability curves labelled P + Q = 0.2 or f + g = 0.13 in Figure 22 that the stability of the pavement under the loaded area increases quite rapidly with distance inward from the edge. This improvement in stability with increasing distance inward from the edge is still more rapid for larger corresponding values of P + Q = 0.5 and f + g = 0.325, P + Q = 1 and f + g = 0.65, P + Q = 2 and f + g = 1.3), as illustrated by the respective stability curves in Figure 22.

Figure 23 is similar to Figure 22, with the exception that the pavement stability even at the edge, about 173 p.s.i., is considerably greater than the vertical load V = 100 p.s.i. to be carried. This represents a condition of overdesign. The stability curves for all values of P + Q and f + g are steeper in Figure 23 than in Figure 22.

Figure 24 represents a condition of pavement under design for edge conditions, since the pavement stability is only about 63 p.s.i. at the edge, whereas the applied load V = 100 p.s.i. The stability curve labelled P + Q = 0.2, or f + g = 0.087, indicates that even for the amount of frictional resistance between pavement and tire and between pavement and base represented by P + Q = 0.2 (equations (9) and (11), or by f + g = 0.087, only at a very considerable distance inward from the edge does the pavement develop stability equal to the applied load V.

Figure 25 has reference to pavement stability for the much smaller contact area of truck tires. With respect to the relationships between the stability curves for values of P + Q varying from 0 to 2, or of f + g varying from 0 to 1.30, it is quite similar

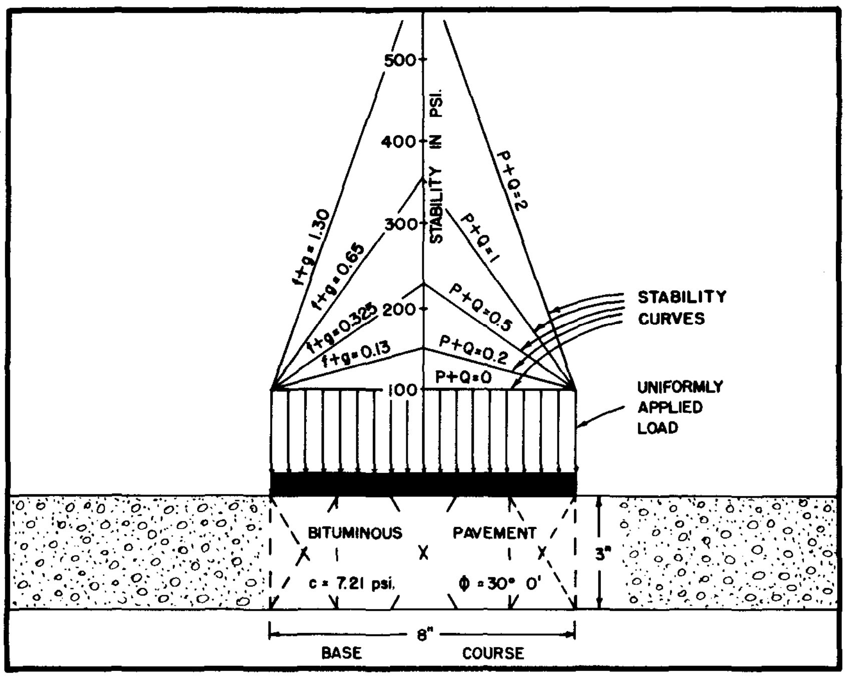



Figure 25. Relationships between Applied Load and Stability of Bituminous Pavements at Varying Distances From Edge Under the Loaded Area and for Different Degrees of Frictional Resistance Developed between Pavement and Tire and between Pavement and Base. (Pavement Stability Equal to Applied Load for Edge Conditions) Truck Tire

to Figure 22 for the larger contact area of large aeroplane tires.

The important conclusion to be drawn from Figures 22, 23, 24, and 25, is that for a uniformly applied load, the portion of the bituminous pavement just under the edge of the loaded area is the most critical insofar as the stability of bituminous mixtures is concerned. At any distance inward from the edge, the bituminous pavement under the loaded area tends to develop increased stability, if there is frictional resistance between pavement and tire and between pavement and base, the actual increase in stability depending upon the distance from the edge, and upon the value of the frictional resistance between pavement and tire and between pavement and base, represented by $P + Q$ or $f + g$.

Since Figures 22, 23, 24, and 25 indicate that whenever frictional resistance is developed between pavement and tire and between pavement and base, pavement stability increases with increasing distance inward from the edge of a uniformly loaded contact area, it seems not unreasonable to ask the following two questions:

(a) Is it necessary that the stability of the pavement be designed to be equal to the applied load at every point on the contact area?

(b) If the answer to question (a) is negative, then over what percentage of the contact area can underdesign be tolerated without resulting in poor service performance?

The initial reaction of a highway or airport engineer to these two questions will probably be that the pavement should not be underdesigned for any portion of the contact area. However, this reaction results intuitively from an engineer's particular type of training and might not necessarily apply in this case. There are localities and occasions when it would be of considerable economic value to know definitely whether or not a bituminous pavement would give satisfactory service, even though underdesigned for a portion of the contact area. This is of particular importance for pavement design for secondary highways, and for roads in a still lower category, which frequently cannot be economically paved unless local aggregate materials can be employed.

Some pavements appear to have been designed, or at least constructed on this basis in the past. They show some distortion under traffic, particularly in hot weather, but have given reasonably satisfactory service, apparently because normal traffic distribution tends to keep them ironed out.

If a certain ratio of overstressed to understressed pavement under the contact area can be permitted, e.g. Figure 24, it would be reasonable to expect that for the same uniform contact pressure, and all other factors being equal, a given bituminous paving mixture might develop adequate stability on the paved area of an airport under wide aeroplane tires, but be quite unstable under the much narrower truck tires, if used for paving a highway. The ratio of understressed to overstressed pavement under the contact area in this case would be much greater for the aeroplane than for the truck tire.

There is insufficient existing information to adequately answer the two questions that have been raised in connection with Figures 22, 23, 24, and 25, and they must, therefore, remain open for the time being. Nevertheless, they are questions that merit sufficient experimental work to provide satisfactory answers, in view of the ever increasing demand for paved roads and highways and the economic necessity for maximum utilization of local aggregate materials when attempting to meet this demand.

Since bituminous paving mixtures apparently develop their lowest stability near the edge of the contact area, the conditions of stability across the first element just within the edge of the loaded area should be examined. These are illustrated in Figure 26 for uniform loading on the contact area. The exact location of this first element is shown in Figure 26(a). The relationship between b, t, and the potential angle of failure, $45 - \frac{\phi}{2}$, for this element, is given by equation (13). The symbol n refers to the number of the element of width b under consideration, with the numbering beginning from the edge of the loaded area, Figure 21. Thus n = 1 for the first element just within the edge of the contact area, n = 2 for the second element, etc. It is apparent that n can also have fractional values.

Figure 26(b) illustrates the value of $L_R$ distributed uniformly across the vertical face of depth t of element (1), (n = 1), that is equivalent to the frictional resistance fV acting over the width b between pavement and tire, plus gV acting over width b between pavement and base. For the fraction of a unit element represented by n = 1/2, Figure 26(c) demonstrates that the width of this element is only b/2. Consequently, the total frictional resistance between pavement and tire and between pavement and base, and therefore $L_R$, is only one half of that for the full unit element, Figure 26(b). Therefore, when n = 1/2, the magnitude of $L_R$ is reduced to one half of the value it had for the full unit element, n = 1. Similarly, Figure 26(d) shows that for the fraction of a unit element represented by n = 1/4, $L_R$ has only one quarter of its value for the full unit element. Finally, when n = 0, which represents the

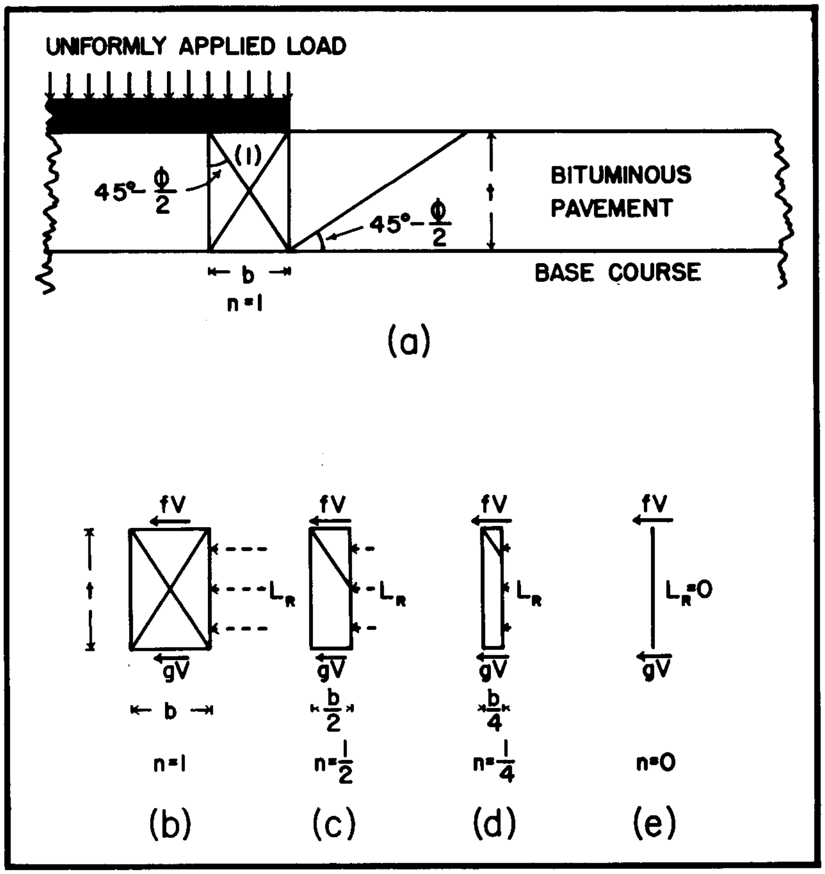



Figure 26. Illustrating, Over the First Element, the Influence of Distance From the Edge on the Magnitude of the Lateral Support $L_R$ Due to Frictional Resistance between Pavement and Tire and Pavement and Base.

vertical plane through the edge of the contact area, b = 0, and therefore $L_R = 0$. Consequently, over the first element within the edge of the loaded area, the value of $L_R$ varies from zero at the edge, to a maximum at a distance b in from the edge, where b is the width of the unit element, n = 1, as illustrated in Figure 26(a).

Since $L_R$ is zero at the exact edge of the contact area, it might seem reasonable to completely disregard the frictional resistance between pavement and tire and between pavement and base as a source of lateral support in the design of bituminous mixtures. This would imply that the only lateral support L available is that provided by the pavement adjacent to the loaded area. A conservative design could be made on this basis, and it would utilize equation (7) as the stability equation to be employed.

Nevertheless, further consideration of the conditions of loading near the edge of the loaded area seem to indicate that some portion of the frictional resistance between pavement and tire and between pavement and base does contribute to pavement stability, and that some value for $L_R$ is, therefore, usually justified for design.

The pressure contour lines over the tire contact area indicated by the investigations of Teller and Buchanan[15], and of Porter[16], demonstrate that there is an area of some width just inside the edge of the contact area, over which the pressure drops from its full average value to zero, e.g. Figure 15. This is particularly true of the rounded tires generally employed for aircraft, and may be somewhat less so for truck and bus tires equipped with much flatter treads. In both of the investigations just referred to, the maximum pressure V on the contact area occurred at some distance in from the edge in a transverse direction across the contact area, and is thought to be due to the stiffness of the side walls of the tires. Therefore, for aeroplane tires in particular, and probably for truck and bus tires with flatter treads as well, the maximum vertical pressure on the pavement is exerted at some distance inward from the edge of the contact area, that is, at some value of n, Figure 26. Between this point and the edge of the loaded area, the contact pressure drops gradually to zero. Consequently, if design should be based upon a critical pressure, V, on the contact area, some value for $L_R$ is developed between the point where this critical pressure occurs and the edge of the loaded area.

The actual distance n, measured in unit elements, Figure 26, from the edge of the loaded area to the point on the contact area at which the vertical pressure V on which design should be based, occurs, probably varies from tire to tire, and with the conditions of loading. The determination of the variation in this distance n from the edge, and of the average value of $L_R$ to be employed over this distance, are matters that require some experimental study.

For the design curves based upon equations (18) and (19) shown in Figures 27 and 28, respectively, to illustrate the influence on pavement stability of frictional resistance between pavement and tire and between pavement and base, it has been assumed that $n = 1$, that is, the critical point of loading is at a width of one unit

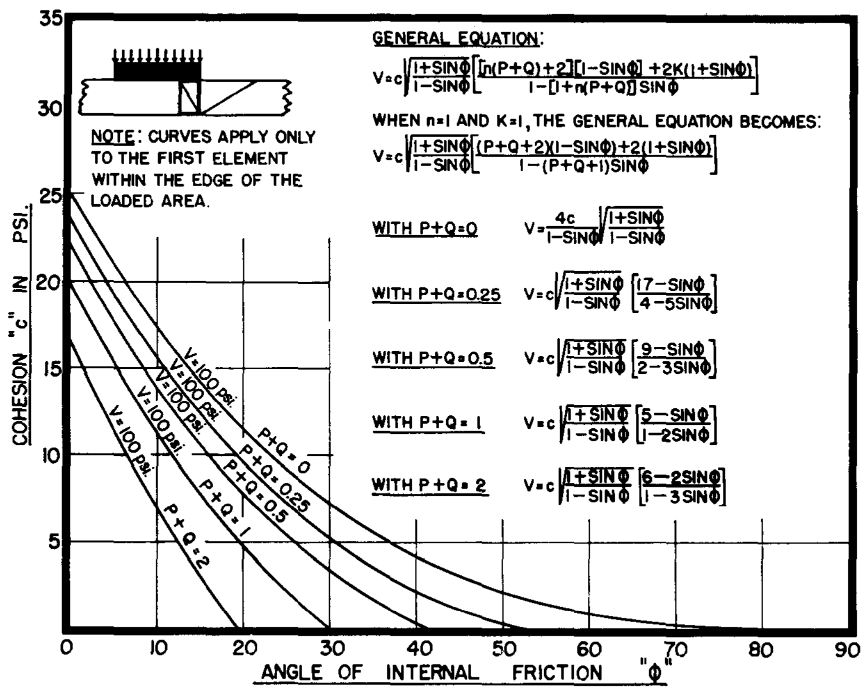



Figure 27. Influence of Frictional Resistance between Pavement and Tire and between Pavement and Base on Design of Bituminous Mixtures to Carry a Specific Load (P + Q Values)

element within the edge of the loaded area, Figure 26, and that the load is uniformly distributed over the contact area. This leads to a somewhat smaller design load than the maximum that may actually occur on the loaded area, and to a somewhat larger value of $L_R$ than may be really developed. It should be noted, however, that the value of $L_R$ to be utilized for pavement design for any project, can be modified as required by adjusting the value of P + Q or f + g to be employed. In addition, the value of K = 1 has been taken for the design curves shown in Figures 27 and 28 and, as pointed out in an earlier section, this value of K seems to be quite conservative.

Based on uniform pressure distribution on the contact area, the curves in Figure 27 illustrate the important influence of frictional resistance between pavement and tire and between pavement and base on the design of bituminous paving mixtures. Each curve indicates the minimum values of c and ø required to carry

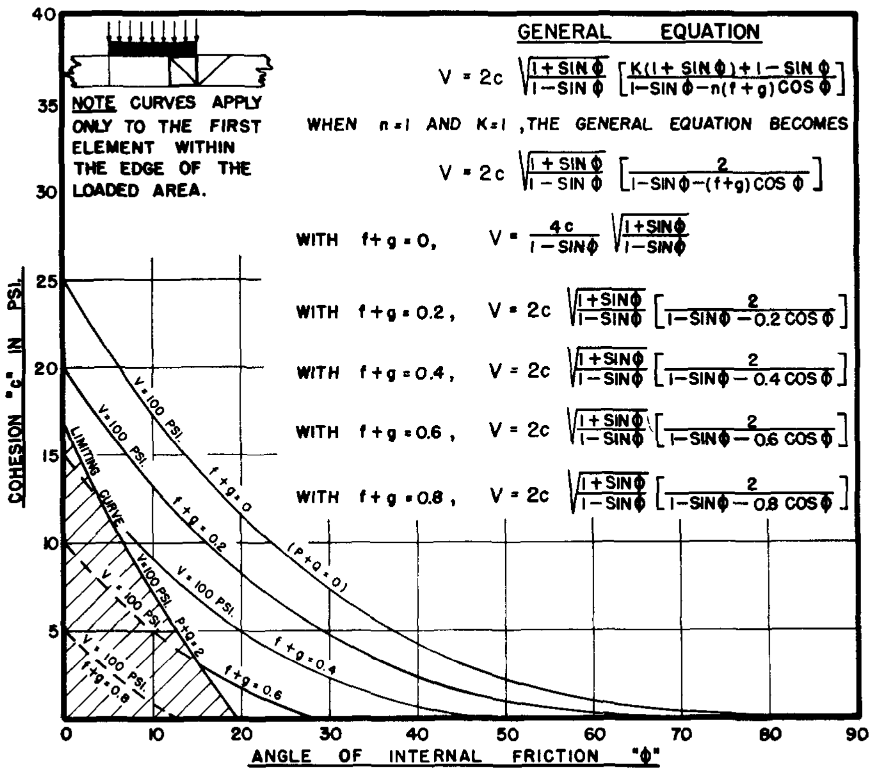



Figure 28. Influence of Frictional Resistance between Pavement and Tire and between Pavement and Base on Design of Bituminous Mixtures to Carry a Specific Load (f + g Values)

a vertical load V of 100 p.s.i., under the conditions assumed, as this frictional resistance is gradually increased from zero, P + Q = 0, to its maximum value, P + Q = 2. For example, if the cohesion c is 5 p.s.i. in each case, an angle of internal friction Ø of about 37° is required to carry this vertical load when P + Q = 0. Ø decreases to about 31° when P + Q = 0.25, to about 26° when P + Q = 0.5, to about 19.5° when P + Q = 1, and to about 13° when P + Q = 2.

Figure 28 is similar to Figure 27, but is based upon f + g values. The curves of Figure 28 (f + g values) have a flatter slope than those of Figure 27 (P + Q values). Of particular interest are the broken line portions of the stability curves for f + g = 0.4, f + g = 0.6, and f + g = 0.8, in the cross-hatched area in the lower left hand corner of Figure 28. The curve labelled $P + Q = 2$ indicates the maximum values of $f + g$ and the corresponding minimum values of c and $\phi$ that could be utilized for the design of stable bituminous paving mixtures for the conditions represented by Figure 28. Consequently, the broken line portions of the curves for $f + g = 0.4$, $f + g = 0.6$ and $f + g = 0.8$ within the cross-hatched area represent imaginary, rather than real conditions of design. They are imaginary because they lie within the part of Figure 28 for which the value of either P or Q, or both, would be greater than unity. It was pointed out earlier in connection with equations (9) and (11), that neither P nor Q can have a value greater than unity, since this would mean that either the frictional resistance between pavement and tire fV, or between pavement and base gV, or both, could be greater than the shearing resistance of the pavement itself. Therefore, Figure 28 emphasizes the need for making use of the limiting $P + Q = 2$ curve to avoid the possibility of an unstable design that could result from employing $f + g$ curves alone.

Figure 29 contains a series of design curves based upon equation (19) for values of applied vertical load V varying from 40 to 400 p.s.i., assuming that $n = 1$, $K = 1$, and $f + g = 0.2$. It should be clear that stability diagrams similar to Figure 28, and based upon equation (19), can be drafted for other values of n, K, and $f + g$. Only those bituminous mixtures with combinations of c and $\phi$ lying on or to the right of any given stability curve would be stable under the vertical load V indicated for that curve.

By comparing Figure 29 with Figure 14 ($K = 1$ for both figures), the importance of frictional resistance between pavement and tire and between pavement and base on the design of bituminous paving mixtures for any applied vertical load V can be observed. For a vertical load $V = 200$ p.s.i., for example, if the bituminous mixture develops a value for cohesion $c = 10$ p.s.i. in both cases, Figure 14 indicates that the corresponding required value of $\phi = 37.5^{\circ}$, while for Figure 29 the required value of $\phi$ is only about $28.5^{\circ}$. If, on the other hand, the bituminous mixture has an angle of internal friction $\phi = 30^{\circ}$ in both cases, Figure 14 indicates that the corresponding required value for cohesion $c = 15$ p.s.i., while for Figure 29 the required value for c is only about 9 p.s.i. These marked differences in the design requirements of Figures 14 and 28 for the same unit load reflect the importance of the influence of even the moderately developed frictional resistance between pavement and tire and between pavement and base, represented by $f + g = 0.2$

In Figures 27, 28, and 29, it will be observed that the design curves cross the abscissa at values of $\phi$ within the normal range

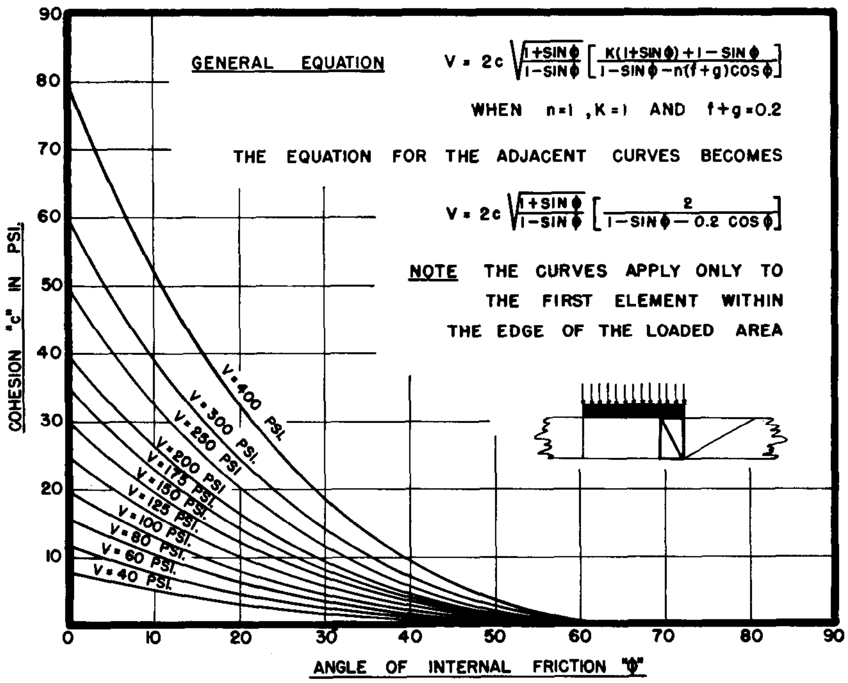



Figure 29. Design Chart for Bituminous Mixtures Based on Triaxial Compression Test (f + g = 0.2)

of those employed for bituminous paving mixtures, that is, cohesion c becomes zero at these values for Ø. Since bituminous mixtures require some cohesion to resist the stresses of traffic, it may be necessary after an experimental study of bituminous pavements of different service performance, to arbitrarily assign a minimum value of cohesion c, c = 5 p.s.i. for example, to all design charts similar to those of Figures 14, 17, 27, 28, 29, etc. However, as previously pointed out, experience may show that any arbitrarily established minimum value of cohesion c should vary with both Ø and V.

It can be demonstrated that the frictional resistance between pavement and tire and between pavement and base provides a reasonable explanation for the well known fact that the stability developed by a given bituminous paving mixture in the field is materially influenced by the thickness of the pavement in which it is incorporated. Figure 30 is employed for this purpose.

To simplify the calculation of frictional resistance between

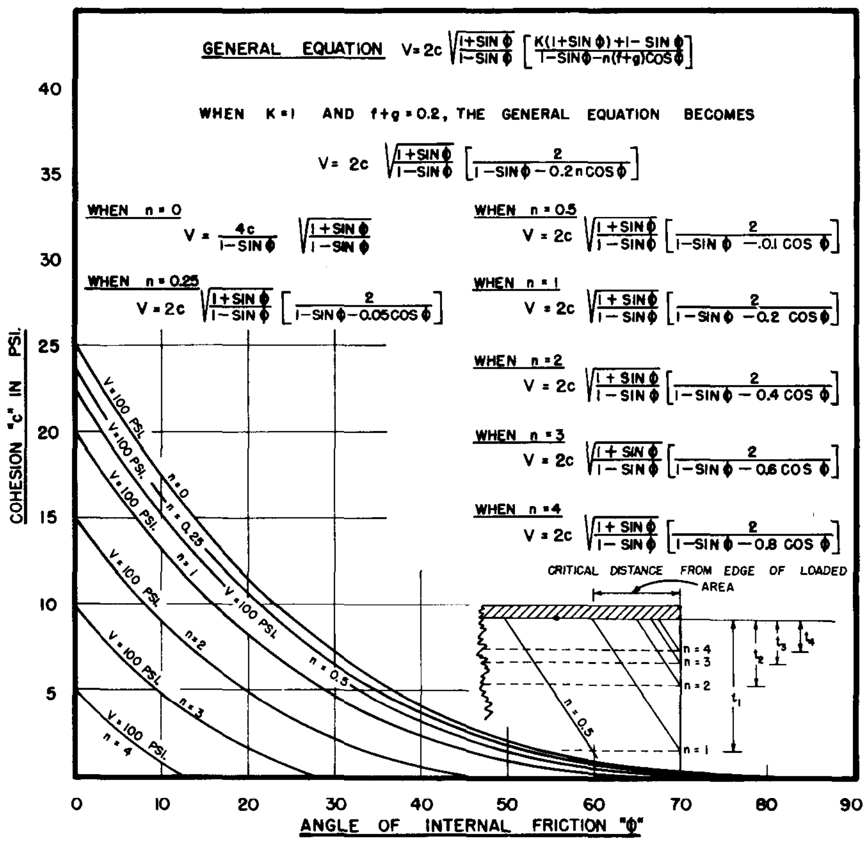



Figure 30. Influence of Thickness on the Stability of Bituminous Pavements with f + g = 0.2 (Does Not Apply to Pavements Subject to Severe Braking Stresses)

pavement and tire and between pavement and base, it is assumed for Figure 30 that a vertical load V = 100 p.s.i. is applied uniformly over the entire contact area. However, as previously pointed out, the work of Teller and Buchanan[15], and of Porter[16], has demonstrated that the pressure on the tire contact area drops from its maximum value to zero over a relatively narrow distance within the edge of the loaded area. For Figure 30, it is assumed that the critical pressure on which design should be based, V = 100 p.s.i., is applied at a critical distance inward from the edge of the contact area as shown. This critical distance from the edge of the contact area can be measured in terms of the number "n" of unit elements between the edge and the point of application of the critical load, V = 100 p.s.i. Figure 21 and equation (13) show that the width b of each unit element depends upon the thickness of the pavement t, and the angle of internal friction $\phi$ of the bituminous paving mixture. Consequently, the number n of unit elements between the edge of the contact area and the point of application of the critical load provides an indirect relationship or ratio between this critical distance and the pavement thickness. The small diagram in the lower right hand corner of Figure 30 demonstrates that as the thickness of pavement is continuously increased, the critical distance from the edge of the loaded area is successively equal to the width of 4 elements, that is n = 4; of 3 elements, when n = 3; of 2 elements, n = 2; of 1 element, n = 1; of one half element, n = 0.5, etc. Finally, if the pavement were of infinite thickness, the critical distance would be equivalent to n = 0.

The influence of pavement thickness on pavement stability is illustrated by the curves of Figure 30, all of which are based upon equation (19) for a uniformly applied load V = 100 p.s.i., K = 1, and f + g = 0.2. The curves demonstrate that to carry this vertical load V = 100 p.s.i., the most stable bituminous mixture (highest corresponding values of c and $\phi$) is required for a pavement of infinite thickness, n = 0, and that mixtures of successively smaller stabilities (lower corresponding values for c and $\phi$) can be employed, as the thickness of pavement is decreased.

The data of Table 1 were obtained from Figure 30 by assuming that the critical distance from the edge of the contact area is 1 inch. The thickness data of Table 1 were computed by substituting appropriate values from Figure 20 in equation (13). Figure 31 represents the data of Table 1 in graphical form. Both Table 1 and Figure 31 indicate that when cohesion c is kept constant, the angle of internal friction $\phi$ of the bituminous mixture must be increased as the pavement thickness t is increased. It should be

Table 1. Relationships between Thickness t and Angle of Internal Friction $\phi$ for Constant Values of Cohesion c, When Critical Distance from the Edge of the Loaded Area is 1 Inch, and for the Conditions Illustrated in Figure 30

| Value of n | Corresponding Values for $\phi$ and t when | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | c = 0 p.s.i. | | c = 5 p.s.i. | | c = 10 p.s.i. | | c = 15 p.s.i. | | c = 20 p.s.i. | |
| | $\phi$° | t in. | $\phi$° | t in. | $\phi$° | t in. | $\phi$° | t in. | $\phi$° | t in. |
| 0 | 90 | $\infty$ | 31 | $\infty$ | 23 | $\infty$ | 14 | $\infty$ | 6 | $\infty$ |
| 0.25 | 84 | 76.4 | 35 | 7.72 | 21°30' | 5.96 | 12 | 4.84 | 4°30' | 4.24 |
| 0.5 | 79 | 21.0 | 33 | 3.72 | 19°30' | 2.80 | 10 | 2.38 | 3 | 2.04 |
| 1.0 | 67 | 4.91 | 29 | 1.70 | 16 | 1.32 | 7 | 1.11 | 0 | 1.00 |
| 2.0 | 46 | 1.25 | 20 | 0.72 | 8 | 0.57 | 0 | 0.5 | | |
| 3.0 | 28 | 0.56 | 9°30' | 0.39 | 0 | 0.33 | | | | |
| 4.0 | 13 | 0.31 | 0 | 0.25 | | | | | | |
| 5.0 | 0 | 0.20 | | | | | | | | |

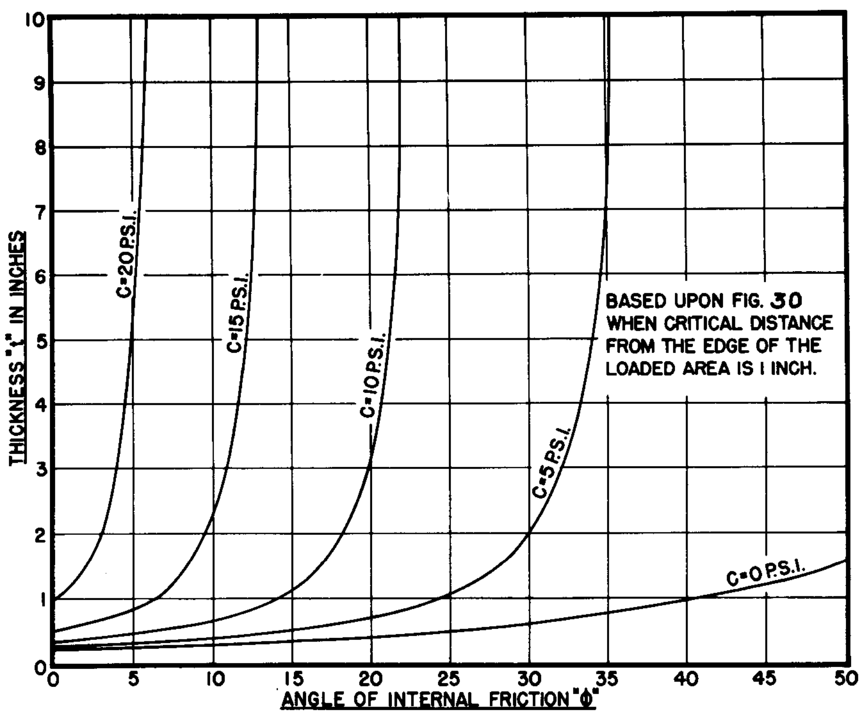



Figure 31. Illustrating Relationships between Thickness t, Cohesion c and Angle of Internal Friction Ø for Bituminous Pavement Design (Does Not Apply to Pavements Subject to Severe Braking Stresses)

apparent that information similar to that of Table 1 and Figure 31 could be prepared, showing that for constant values of angle of internal friction Ø the cohesion c of the bituminous mixture would have to be increased as pavement thickness t was increased.

Figure 32 was prepared from the data of Figure 31. To support a given vertical load V = 100 p.s.i., Figure 32 indicates clearly that paving mixtures of considerably greater stability (higher corresponding values of c and Ø) are required when pavement thickness t is large, than when it is small. For example, compare the values of c and Ø required by the stability curve for a pavement thickness t = 10 inches, with those for the curve for t = 0.25 inch.

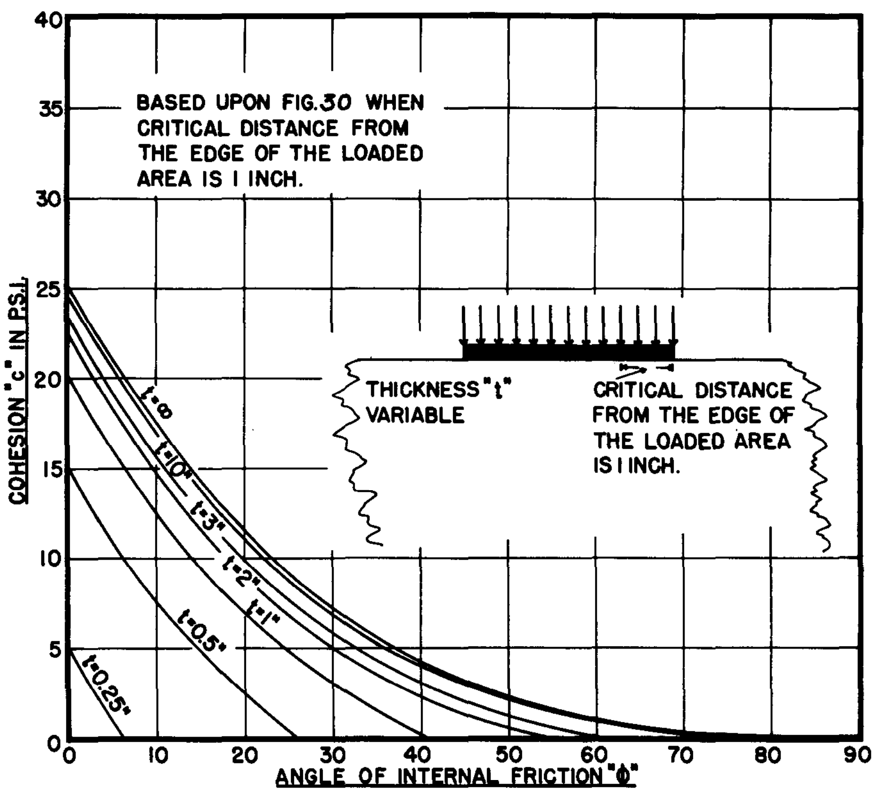



Figure 32. Illustrating the Influence of Pavement Thickness on the Design Requirements for Bituminous Pavements (Does Not Apply to Pavements Subject to Severe Braking Stresses)

The information provided by Table 1 and Figures 30, 31, and 32, is quite in keeping, at least qualitatively, with practical experience and observation. Properly constructed surface treatments, for example, including those made with sand cover material, in spite of their relatively high binder content, show little tendency to be squeezed out from under even narrow, high pressure, truck tires. It is also well known that a bituminous mixture, which shows questionable or insufficient stability when laid as a thick pavement, may develop quite adequate stability when placed in a relatively thin layer.

Finally, it should be observed in connection with Figures 28, 30, 31, and 32, Table 1, and equation (19), that, if there were no frictional resistance between pavement and tire and between pavement and base, that is, $f + g = 0$, the stability of bituminous pavements would be independent of their thickness, which is contrary to practical experience. It might also be noted that when $P + Q = 0$, or $f + g = 0$, equations (18) and (19), respectively, revert to equation (7), which does not include the influence of frictional resistance between pavement and tire and between pavement and base.

So far in this section it has been assumed that the tire pressure is applied uniformly over the entire contact area, when investigating the influence of frictional resistance between pavement and tire, fV, and between pavement and base, gV, on the stability of bituminous paving mixtures. This has been done to avoid dealing with too many variables at a time.

Reference has already been made to the investigations by Teller and Buchanan[15] and by Porter[16], which have demonstrated that the tire pressure is not uniformly distributed over the contact area, but varies from zero at the edge to a maximum at some distance inward from the edge, e.g. Figure 15. It is worth while to determine what influence this variable distribution of tire pressure on the contact area may have on the design of stable bituminous paving mixtures, insofar as it is affected by frictional resistance between pavement and tire and between pavement and base.

In Figure 33, the shape of the pressure distribution curve across the transverse axis of the contact area of a truck tire is similar to (but not identical with) that actually determined by Teller and Buchanan[15]. The short curves on the right and left hand sides represent stability curves for paving mixtures having the specific values of c and $\emptyset$ indicated on the left hand side. The stability curves on either side demonstrate the increase in pavement stability that occurs with increasing distance inward from the edge of the contact area, for different values of $f + g$, and for $P + Q = 2$. It will be recalled that $P + Q = 2$ establishes the maximum values permissible for $f + g$ in actual pavement design for stationary loads, and for wheel loads moving at a relatively uniform rate of speed.

The stability curves of Figure 33 were determined from modifications of equations (20) and (21), respectively,

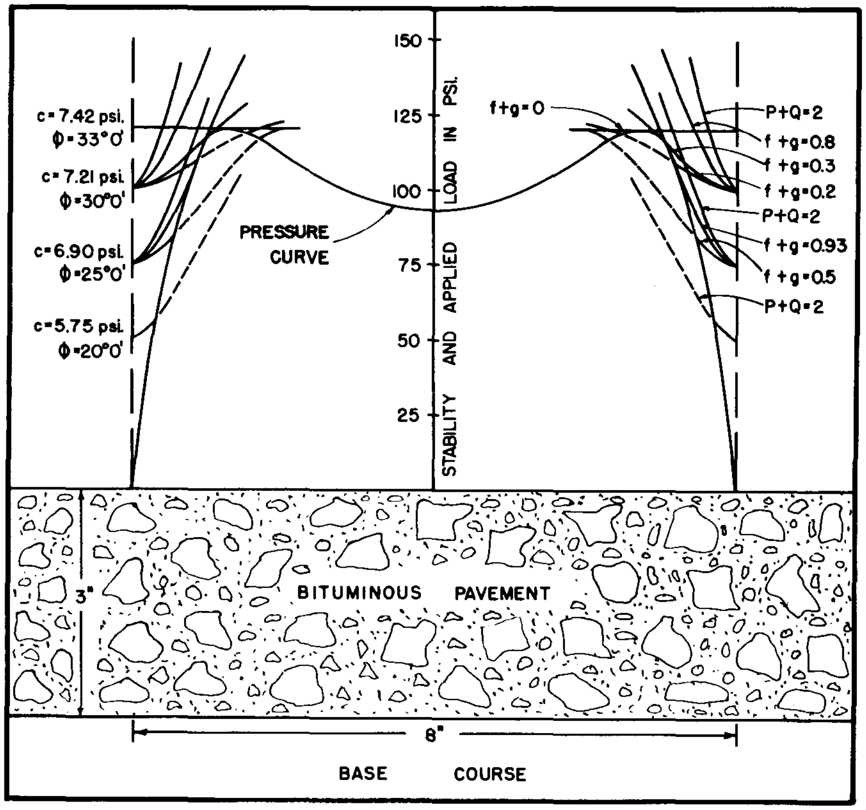



Figure 33. Influence of Typical Pressure Distribution Over the Contact Area, and of Various Degrees of Frictional Resistance between Pavement and Tire and between Pavement and Base in Terms of f + g Values on the Design of the Underlying Bituminous Pavement. (Truck Tire)

$$V = 2cK\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}} + n\,(P+Q)\,(c + V''\tan\phi)\left(\tan\left(45 - \frac{\phi}{2}\right)\right)\left(\frac{1+\sin\phi}{1-\sin\phi}\right) \quad (22)$$

and

$$V = 2cK\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\left(\frac{1+\sin\phi}{1-\sin\phi}\right) + 2c\sqrt{\frac{1+\sin\phi}{1-\sin\phi}} + nV''\,(f+g)\left(\tan\left(45 - \frac{\phi}{2}\right)\right)\left(\frac{1+\sin\phi}{1-\sin\phi}\right) \quad (23)$$

where $V$ = the stability developed by the bituminous pavement at any point on the contact area

$V''$ = the average vertical pressure between the edge and the point on the contact area at which the value of stability $V$ is required

and the other symbols have the significance previously defined for them.

If it is an essential requirement of design that the pavement must not be overloaded at any point on the contact area, this is equivalent to stating that pavement stability must be not less than the applied pressure at any point on the loaded area. The critical stability curve, therefore, is the one that is just tangent to the pressure distribution curve. Any stability curve that crosses the pressure curve represents conditions of pavement design that result in overloading of the portion of the contact area for which the stability curve is below the pressure curve. For example, for the conditions of design represented by Figure 33 ($K = 1$ and $t = 3$ inches), and for a paving mixture for which $c = 6.9$ p.s.i. and $\phi = 25^0$, the stability curve for $f + g = 0.93$ is just tangent to the pressure curve, while the stability curve for $f + g = 0.5$ cuts through the pressure curve. Consequently, for this particular paving mixture, $f + g = 0.93$ represents the smallest value of $f + g$ that will prevent the contact area from being overloaded at any point and, therefore, also represents the lowest value of $f + g$ that should be considered for a stable design based upon this particular paving mixture ($c = 6.9$ p.s.i., $\phi = 25^0$) and upon the particular pressure distribution curve and other conditions of design represented in Figure 33.

In Figure 34, the pressure distribution curve shown represents smoothed out data for the variable pressure across the transverse axis of the contact area for a load of 200,000 pounds on a single aeroplane tire, as measured by Porter[16]. The information illustrated in Figure 34 for an aeroplane tire is generally similar to that of Figure 33 for a truck tire.

In Figure 35, the pressure distribution curve is identical with that of Figure 34. For the condition that $K = 1$ and thickness $t$ = 3 inches, Figure 35 illustrates the various values of $c$ and $\phi$ for $f + g = 0.2$ on the right hand side, and for $P + Q = 2$ on the left hand side, that bituminous mixtures must have in order that the resulting stability curves will be just tangent to the pressure curve. It also demonstrates that if either $f + g = 0$, or $P + Q = 0$ (no frictional resistance between pavement and tire and between pavement and base), pavement design must be based upon the maximum pressure

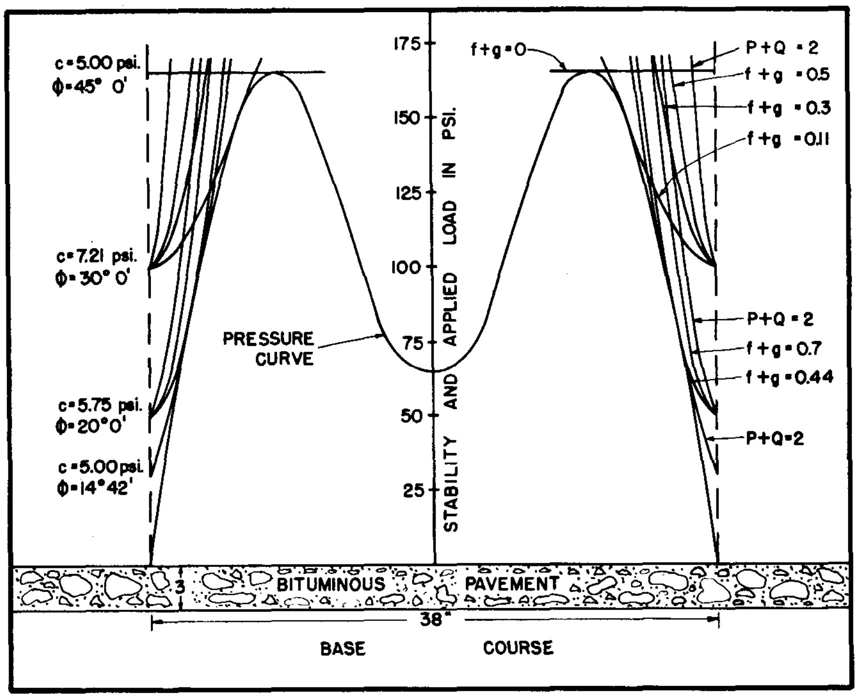



Figure 34. Influence of Typical Pressure Distribution Over the Contact Area, and of Various Degrees of Frictional Resistance between Pavement and Tire and between Pavement and Base in Terms of f + g Values on the Design of the Underlying Bituminous Pavement. (Aeroplane Tire)

applied to the contact area, that is, for V = 165 p.s.i. On the other hand, bituminous mixtures, for example, for which c = 14.3 p.s.i. and $\emptyset = 15^{o}$, and for which c = 3.5 p.s.i. and $\emptyset = 34^{o}$, that in the complete absence of frictional resistance between pavement and tire and between pavement and base would support vertical pressures of only V = 100 p.s.i. and V = 60 p.s.i., respectively, provide stability curves just tangent to the pressure curve, if the value of f + g = 0.2. Figure 35 clearly demonstrates, therefore, the important influence of frictional resistance between pavement and tire and between pavement and base on pavement stability, in making it possible to safely utilize bituminous paving mixtures that would otherwise support only a fraction of the maximum vertical load V on the contact area. Diagrams similar to Figure 35,

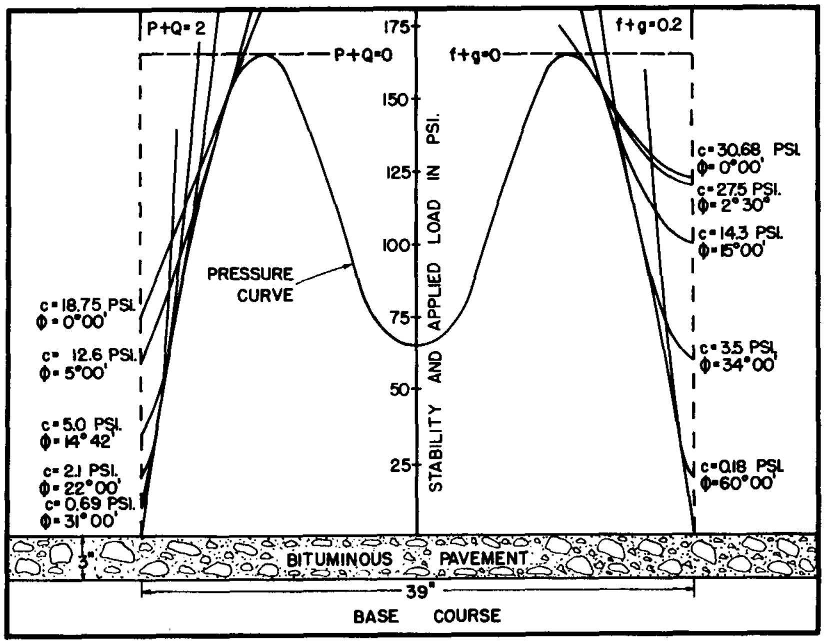



Figure 35. Illustrating the Influence of the Pressure Distribution Curve for the Loaded Area and of Given Values for f + g or P + Q, on Bituminous Pavement Design. (Aeroplane Tire)

demonstrating the influence on pavement design of values for f + g other than 0.2, can be easily prepared.

Figure 36 is a design diagram indicating all the combinations of values for c and $\phi$ that could be selected for each value of f + g for the design of a stable bituminous paving mixture for the conditions of loading represented by the pressure distribution curve in Figure 35, when K = 1 and the pavement thickness t = 3 inches. If f + g = 0, the bituminous mixture must be designed for the maximum vertical load V on the contact area, that is for V = 165 p.s.i. Bituminous mixtures with smaller values for c and $\phi$ can be selected if f + g = 0.2, and still smaller values of c and $\phi$ can be utilized if f + g = 0.6. For example, Figure 36 demonstrates that a bituminous mixture which in the entire absence of frictional resistance between pavement and tire and between pavement and base (f + g = 0) would support a maximum vertical load V of only about 20 p.s.i., e.g. c = about 1.5 p.s.i., $\phi$ = 30°, if designed and

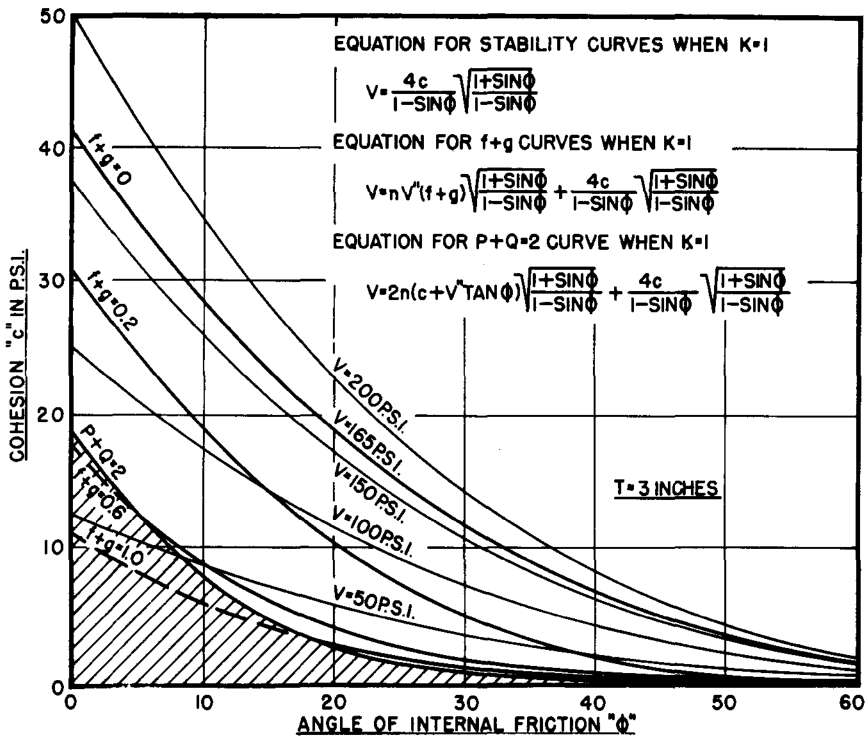



Figure 36. Influence of f + g Values on Bituminous Pavement Design, on the Basis of the Contact Pressure Distribution Pattern of Figure 35.

constructed to have a value for f + g = 1.0, will provide a satisfactory paving mixture for the conditions represented by Figure 35, where the maximum vertical pressure on the contact area is given by V = 165 p.s.i. The stability curves in Figure 36 labelled V = 50 p.s.i., V = 100 p.s.i., V = 150 p.s.i. and V = 200 p.s.i., are identical with the same curves in Figure 14. All the curves in the Figure 14 represent the condition for f + g = 0. The curve labelled P + Q = 2 in Figure 35 indicates the maximum values for f + g and the corresponding minimum values for c and ∅ that could be utilized for the design of a bituminous mixture for the particular pressure distribution curve and other conditions represented in Figure 35. It should be emphasized again that the design diagram of Figure 36 applied only to the particular conditions represented by Figure 35. For every change in the shape of the pressure distribution curve, pavement thickness t, value for K, and any other condition pertaining to Figure 35, a new design diagram similar to Figure 36 would have to be worked out. Since the pressure distribution curves may be reasonably similar for any given combination of wheel load and tire pressure, the number of design diagrams similar to Figure 36 that would be required for design purposes may not be unduly large.

On the basis of the development presented here, Figure 36 emphasizes a very important principle of bituminous pavement design. It shows very clearly that for any given maximum pressure on the contact area, a stable pavement will result from the use of a bituminous mixture of high inherent stability (high corresponding values for c and Ø), together with a low value for f + g, or from

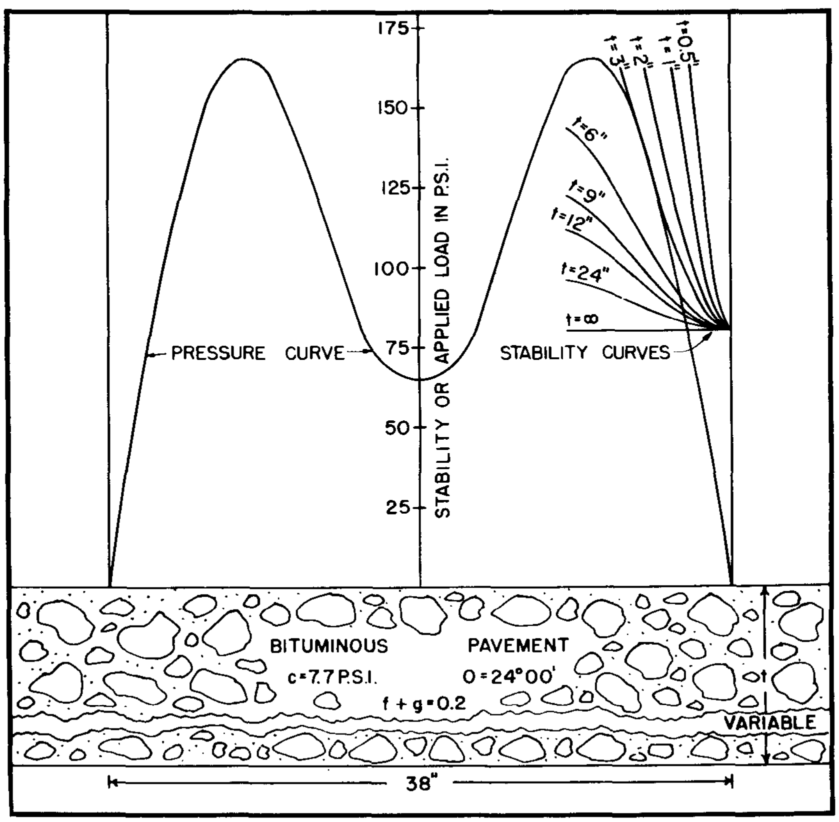



Figure 37(a). Influence of Pavement Thickness on Pavement Stability for Stationary Wheel Loads or Wheel Loads Moving at a Uniform Speed (Aeroplane Tire)

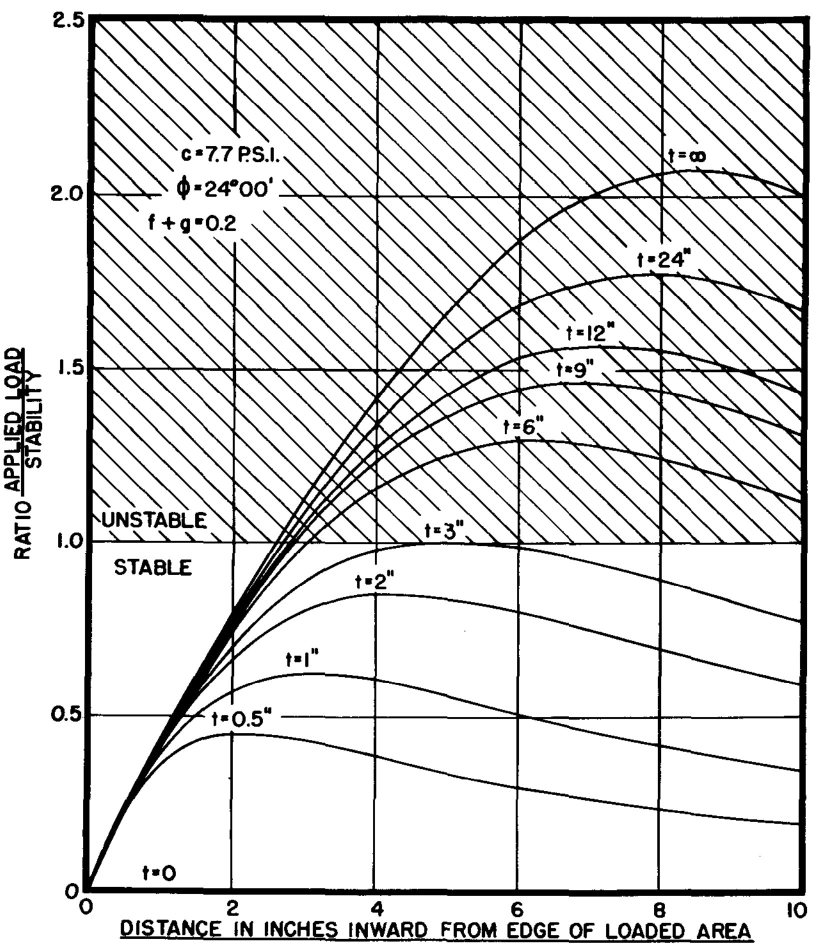



Figure 37(b). Influence of Pavement Thickness on Pavement Stability for Stationary Wheel Loads or Wheel Loads Moving at a Uniform Speed (Aeroplane Tire)

the use of a bituminous mixture of low inherent stability (low corresponding values for c and Ø), combined with a high value for f + g.

Earlier in this section the influence of pavement thickness on pavement stability was discussed for the case of a vertical load V assumed to be applied uniformly over the contact area. The influence of pavement thickness on pavement stability for the non-uniform distribution of vertical load V on the contact area will now be considered, and it is illustrated in Figures 37(a), (b), (c), and (d).

The stability curves shown in the upper right hand corner of Figure 37(a) for different thicknesses of a given paving mixture (c = 7.7 p.s.i. $\phi$ = 24°) are based upon equation (23). Values of K = 1 and f + g = 0.2 were selected for purposes of illustration. The thickness t pertaining to each stability curve can be obtained by reference to equation (11) and Figures 21 and 26. The curve for applied pressure distribution over the contact area is identical with that for Figure 35.

If adequate design requires the pavement to develop sufficient stability that no part of the contact area is overloaded (that is, the stability developed is to be not less than the applied pressure at any point on the contact area), Figure 37(a) demonstrates that the maximum thickness of pavement that can be used for the particular conditions pertaining to this diagram is 3 inches. The stability curve for a pavement thickness of 3 inches is just tangent

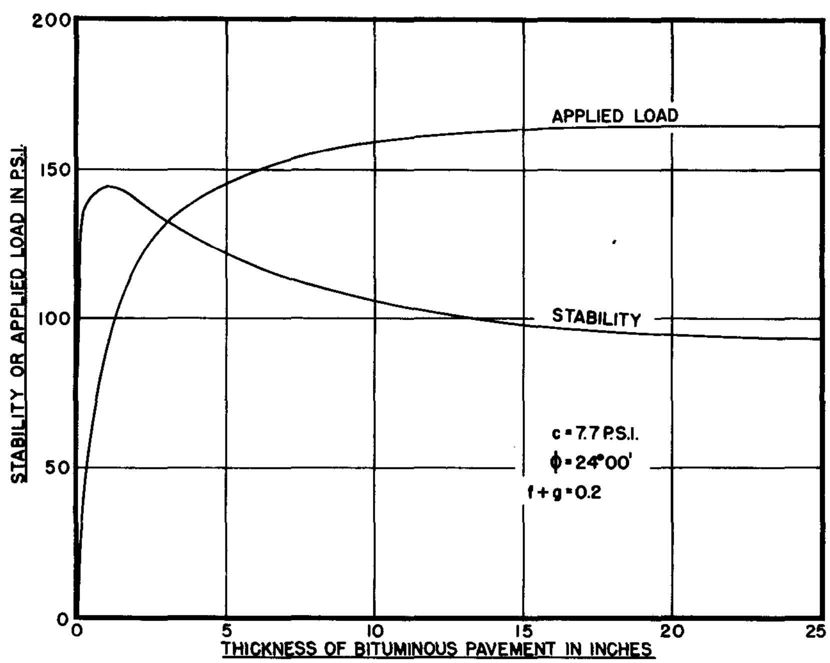



Figure 37(c). Influence of Pavement Thickness on Pavement Stability for Stationary Wheel Loads or Wheel Loads Moving at a Uniform Speed (Aeroplane Tire)

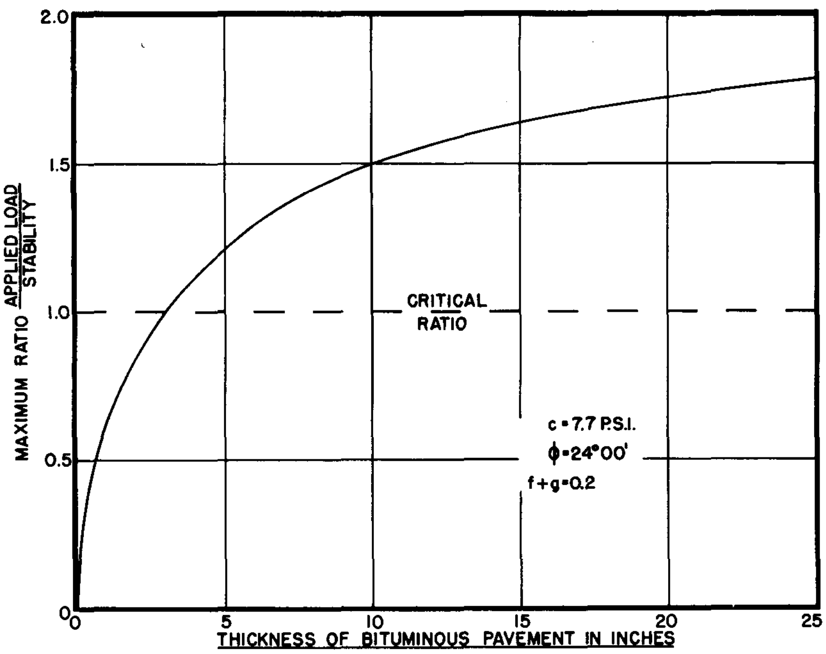



Figure 37(d). Influence of Pavement Thickness on Pavement Stability for Stationary Wheel Loads or Wheel Loads Moving at a Uniform Speed (Aeroplane Tire)

to the pressure distribution curve. The stability curve for a thickness of 6 inches, on the other hand, cuts through the pressure distribution curve and indicates that for this thickness the applied pressure would be greater than the pavement stability over a considerable portion of the contact area. Figures 37(b), (c), and (d), are based upon Figures 37(a) and illustrate the relationship between applied load and stability at increasing distances inward from the edge of the contact area, for different pavement thicknesses, in somewhat different form. Figure 37(d) indicates, as do Figures 37(a), (b) and (c), that applied load is just equal to pavement stability when the pavement thickness is exactly 3 inches (stability curve for thickness of 3 inches just tangent to the pressure distribution curve, Figure 37(a)). It also shows that for a thickness of 5 inches the applied load is about 120 per cent of pavement stability, and for a thickness of 10 inches is about 150 per cent of pavement stability at some point on the contact area.

That is, pavement thicknesses of 5 inches and 10 inches would lead to 20 per cent and 50 per cent underdesign in this particular case. It should be emphasized again that Figures 37(b), (c), and (d) are based entirely upon the particular pressure distribution curve, paving mixture (c = 7.7 p.s.i., Ø = 24°), f + g = 0.2, K = 1, and other conditions illustrated in Figures 37(a). A change in the value of any one of these variables would modify each of the four diagrams shown in Figures 37(a), (b), (c), and (d), and, in general, would indicate that some other thickness than 3 inches would be critical. This is well illustrated in Figure 38, wherein the pressure distribution curve is identical with that shown earlier in Figure 33 for a truck tire, the characteristics of the paving mixture are represented by c = 4.54 p.s.i., Ø = 29°, f + g = 0.6, and K = 1. The stability curves in the upper right hand corner indicate that the stability curve for thickness t = 1.5" is just tangent to the pressure distribution curve. Consequently, if sound design

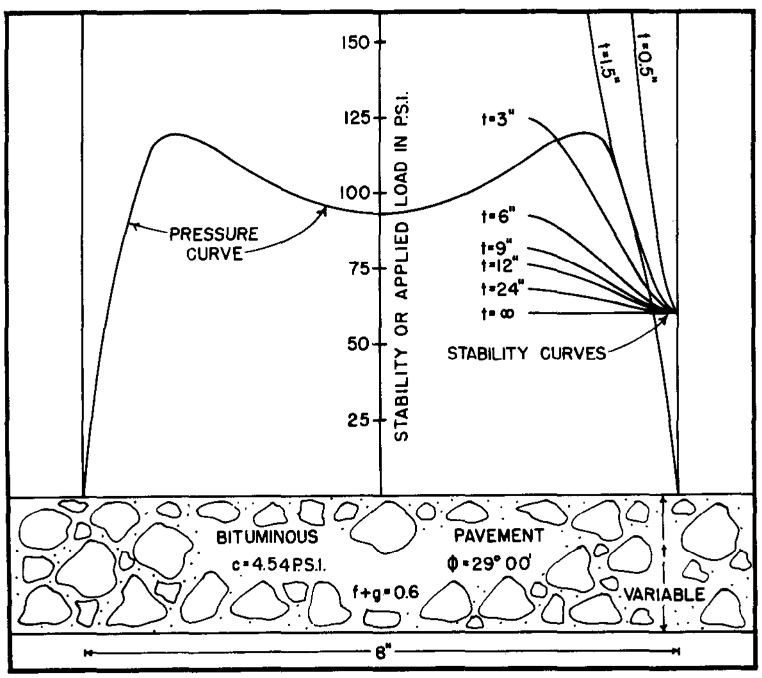



Figure 38. Influence of Pavement Thickness on Pavement Stability for Stationary Wheel Loads or Wheel Loads Moving at a Uniform Speed (Truck Tire)

requires that pavement stability must be not less than the applied load at any point on the contact area, Figure 38 demonstrates that for the given conditions it represents, the maximum thickness of this particular paving mixture that could be utilized is given by $t = 1.5$ in. If any greater thickness of this paving mixture were employed, Figure 38 indicates that pavement stability would be less than the applied pressure at some point on the contact area. If a greater pavement thickness than $t = 1.5$ inches were required, it would be necessary to employ a more stable paving mixture, or to otherwise modify the conditions represented by Figure 38.

The diagrams of Figures 37(a), (b), (c), and (d), and 38, indicate that increased pavement thickness tends to be a liability as far as pavement stability is concerned, when designing for stationary wheel loads, or for wheel loads moving at a uniform rate of speed. That is, to support a given wheel load under these traffic conditions, a thick pavement must be designed to have a higher minimum stability (higher corresponding c and $\emptyset$ values) than a thin pavement, all other factors being equal. As previously pointed out, this is in keeping with practical observations in the field.

## INFLUENCE OF TIRE DESIGN ON THE STABILITY OF BITUMINOUS PAVING MIXTURES

It appears that the design of pneumatic or other tires has been influenced very little by the relationship between tire design and its effect on the performance of the ground surface or pavement on which wheeled vehicles operate. This lack of co-ordination between tire design and its effect on the various surfaces over which tired vehicles move seems to be due to the serious lack of published fundamental information on this subject at the present time.

From the point of view of pavement design and performance, the concept of the influence of frictional resistance between pavement and tire and between pavement and base on the stability of bituminous pavements, that has just been outlined, could apparently be usefully applied to the design of pneumatic or other tires. It has been pointed out in connection with Figures 33 to 38, that the stability of bituminous pavements depends upon the shape of the curve of pressure distribution on the contact area. Since the shape of this pressure distribution curve depends in turn upon the design of the tire, it is apparent that the stability of a bituminous pavement is materially influenced by the designs of the tires on the vehicles that will use the pavement.

This is clearly demonstrated in Figures 39(a), (b) and (c). Figure 39(a) is a diagram of a normal, loaded, pneumatic truck

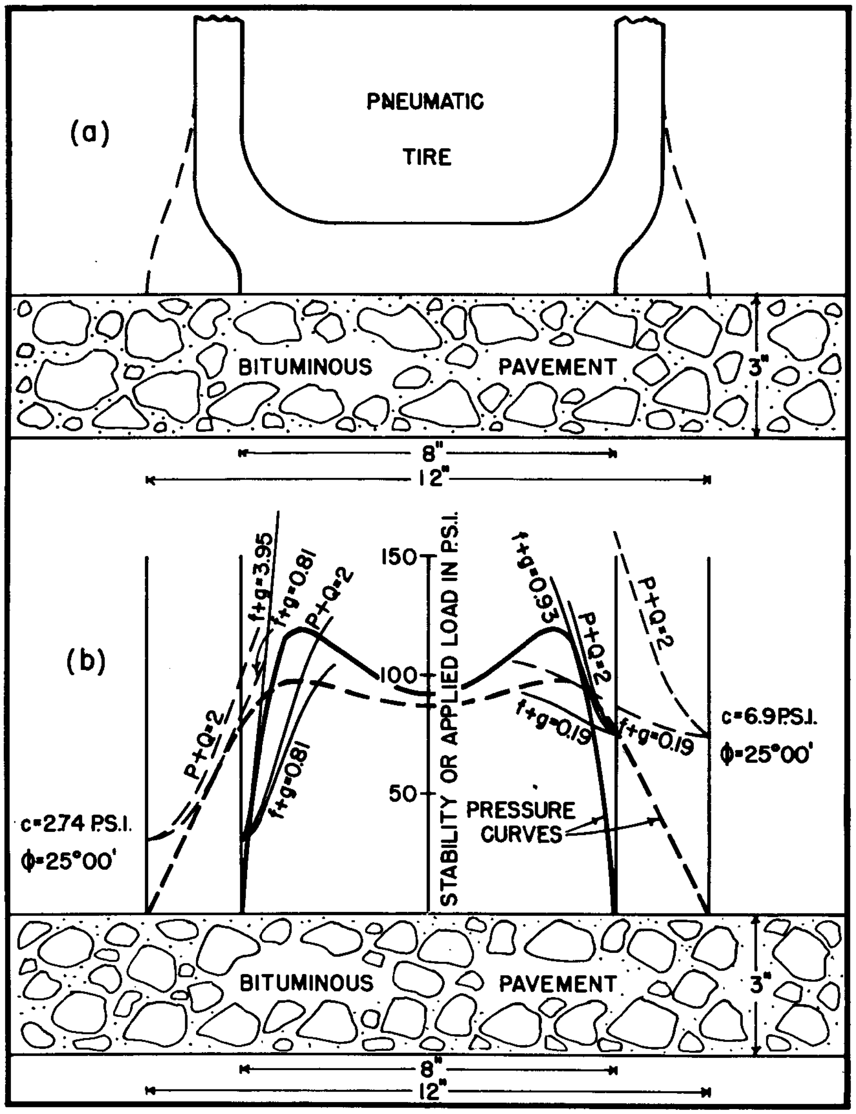



Figure 39. Influence of the Shape of the Curve of Pressure Distribution Across the Contact Area on Bituminous Pavement Design

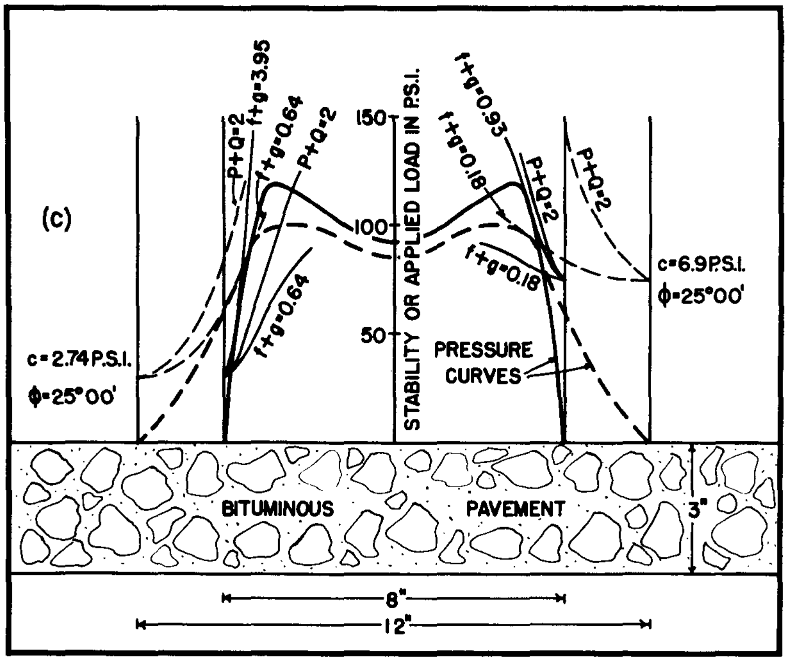



Figure 39. Influence of the Shape of the Curve of Pressure Distribution Across the Contact Area on Bituminous Pavement Design

tire on a pavement. The width of the contact area is 8 inches. The dashed lines on either side indicate that by redesign of the tire, the width of the contact area for the same wheel load could be increased to 12 inches, or to any other desired width.

Figure 39(b) illustrates the probable effect on the shape of the pressure distribution curve, if the width of contact area were increased from 8 to 12 inches for the same wheel load. The heavy solid line represents the general shape of the pressure distribution curve for a normal contact area width of 8 inches, based upon the work of Teller and Buchanan[15]. The heavy broken line represents the shape of the pressure distribution curve that might result if the width of the contact area were increased to 12 inches. One noticeable difference between these two pressure distribution curves is that the maximum pressure ordinate is greater for the solid line than for the broken line curve. Of much greater importance from the point of view of the present discussion, however, is the fact that the slope of the broken line curve between the point of maximum pressure and the edge of the contact area is much flatter than the slope of the same portion of the solid pressure distribution curve.

What this difference between the slopes of the outer portions of these two pressure distribution curves means in terms of bituminous pavement design is clearly indicated by the short stability curves on both the left-hand and right-hand sides of Figure 39(b). The solid stability curves pertain to the solid pressure distribution curve, while the dashed stability curves pertain to the broken pressure distribution curve. On the right hand side, stability curves for a bituminous mixture for which c = 6.9 p.s.i. and $\phi = 25^{\circ}$, and for different values of f + g, are drawn to both pressure distribution curves. It is apparent that for the very small value of f + g = 0.19, the resulting stability curve is just tangent to the broken line pressure distribution curve. On the other hand, the stability curve for f + g = 0.19 cuts through the solid pressure distribution curve and indicates that this particular paving mixture would be unstable for the condition of loading it represents. For the solid pressure distribution curve, f + g = 0.93 is the smallest f + g value that will provide a stability curve just tangent to it for this bituminous mixture. That is, a paving mixture for which c = 6.9 p.s.i. and $\phi = 25^{\circ}$ would provide adequate stability for the loading conditions indicated by the broken line pressure distribution curve if f + g = 0.19, whereas for the loading conditions represented by the much steeper solid pressure distribution curve, f + g = 0.93 is the lowest f + g value that would provide the necessary stability.

The left hand side of Figure 39(b) indicates that a bituminous mixture, for which c = 2.74 p.s.i. and $\phi = 25^{\circ}$, would develop sufficient stability for the loading conditions represented by the broken line pressure distribution curve if f + g = 0.81, since the stability curve for 0.81 is just tangent to the latter pressure distribution curve. On the other hand, a minimum value of f + g = 3.95 would be required to make the same bituminous mixture stable for the conditions of loading given by the solid pressure distribution curve. However, for this particular bituminous mixture, a value of f + g = 3.95 could not be attained, since the stability curve for P + Q = 2 cuts through the solid pressure distribution curve. Since P + Q = 2 represents the highest corresponding value or values of f + g that can be developed, this means that no value of f + g obtainable in actual practice could make this bituminous mixture (c = 2.74 p.s.i., $\phi = 25^{\circ}$) stable under the loading conditions represented by the solid pressure distribution curve.

Consequently, this is an example of a paving mixture that would be stable for a tire design providing the conditions of loading represented by the broken line pressure distribution curve in Figure 39(b), but that would be unstable for a tire design providing the loading conditions indicated by the solid line pressure distribution curve.

Figure 39(c) is similar in many respects to Figure 39(b), but the outer portions of the broken line pressure distribution curve are concave upward, rather than concave downward. The tire redesign leading to the wider contact area might be accomplished in such manner that this concave upward shape for the outer portions of the pressure distribution curve would result. The stability curves on both the right-hand and left-hand sides of Figure 39(c) indicate that this concave upward shape is an asset from the point of view of bituminous pavement design, at least in this case. For example, on the left-hand side, the stability curve for $f + g = 0.64$ for a paving mixture for which $c = 2.74$ p.s.i. and $\emptyset = 25^0$ is just tangent to the broken line pressure distribution curve. For the corresponding stability curve for the same paving mixture in Figure 39(b), a value of $f + g = 0.81$ is required for tangency to the broken line pressure distribution curve. In this case, therefore, adequate stability is provided by $f + g = 0.64$ when the outer portion of the pressure distribution is concave upward, whereas a value of $f + g = 0.81$ is required for the corresponding distribution curve when it is concave downward.

The important conclusion illustrated by Figures 39(a), (b), and (c) is that by flattening the slope of the portion of the pressure distribution curve between the point of maximum pressure and the edge of the contact area, bituminous paving mixtures of lower stability (lower corresponding values of c and $\emptyset$) can be safely employed, than is possible for the pressure distribution curves of tires in normal use today. This is equivalent to stating that redesign of the tires that transmit the wheel load to the pavement, in such a manner that the outer slopes of the pressure distribution curves would be much flatter than they are at the present time, could be utilized very advantageously in the design of the bituminous paving mixtures supporting these wheel loads.

There would seem to be a practical limit to this flattening of the outer slopes of the pressure distribution curves in a transverse direction (widening of the contact area), that should be pointed out. Beyond a certain point, further flattening would provide the pavement with more stability in the transverse than in the longitudinal direction. That is, beyond this point there would be more tendency for a loaded tire to squeeze out the underlying pavement in a longitudinal than in a transverse direction. Consequently, flattening the slope of the outer portions of the pressure distribution curve beyond the critical slope where this occurs would probably be of little practical value.

While it has been demonstrated here that modifications in tire design, that would result in flatter slopes for the outer portions of the corresponding pressure distribution curves, could be utilized very advantageously in the design of bituminous paving mixtures, it is believed that the same principles would be of value for bettering the performance under traffic of either improved or unimproved ground surfaces over which various types of tired vehicles must operate. This applies to automobiles, trucks, aircraft, tractors, earth movers, and tire mounted equipment of all kinds, moving over either hard or soft ground surfaces.

Traffic performance over sand and soft earth, over earth surfaces that have been softened by rain or other moisture, etc., for example, could undoubtedly be very materially improved by the use of tires that have been designed on the basis of the principles outlined here. Improvements in this direction would be of considerable economic value in the agricultural, construction, transportation, lumbering, etc., industries, because of the greater mobility of tired equipment they would make possible. The possibilities for improvement in this direction are sufficiently promising, that the rubber industry and other organizations interested in the greater mobility of equipment, might very profitably devote some research to it.

Incidentally, Figure 39(b) demonstrates why a bituminous pavement tends to be more stable in the longitudinal than in the transverse direction of the contact area under present tires. This was referred to in an earlier section of the paper. Figure 15 prepared from the data of Teller and Buchanan[15] indicates that the slope of the curve of pressure distribution is flatter in the longitudinal than in the transverse direction of the area of contact. Figure 39(b) demonstrates in turn that this flatter slope provides the pavement with greater stability in the longitudinal than in the transverse direction of the loaded area.

## INFLUENCE OF BRAKING STRESSES

In the development outlined so far, the stability of bituminous pavements under stationary wheel loads, or under wheel loads moving at relatively uniform rates of speed, has been considered. Under these conditions of traffic, as previously pointed out, there is a greater tendency for a bituminous pavement to be squeezed out from under a tire in a transverse than in a longitudinal direction.

In actual service, however, in addition to the tendency to be squeezed from under the wheel at right angles to the direction of travel, bituminous pavements are subjected to direct braking and acceleration stresses and to the equivalent of these whenever the moving vehicle changes direction in a vertical or horizontal plane, or both, without braking or acceleration. Braking and acceleration forces are usually applied in the direction of travel and they therefore tend to shove the pavement either ahead of (for braking), or behind (for acceleration) the wheel.

Observation of the pavement at bus stops, at traffic lights, at taxi stands on the street, etc., where there is much stopping and starting of traffic will sometimes show that the pavement has been shoved into a number of waves in a longitudinal direction, with little or no displacement in the transverse direction. This demonstrates that braking and acceleration stresses will sometimes provide the most critical conditions of design for bituminous pavements carrying moving loads. Consequently, the influence of these braking and acceleration stresses on the design of bituminous pavements should be considered in quantitative manner if possible. From their very nature, it is probable that braking stresses are generally more severe than acceleration stresses.

Figure 40 illustrates that forces to be considered for the design of a bituminous pavement capable of resisting braking and acceleration stresses. To simplify the approach to this problem, the contact area between tire and pavement is assumed to be rectangular in shape. Although the contact areas for aeroplane tires are elliptical in shape, Paxson's[17] measurements indicate that the tire contact areas of trucks and buses are very nearly rectangular. If the area of contact is assumed to be rectangular, the element of pavement involved, when considering the influence of braking or acceleration stresses, is the rectangular block of pavement, abcdjihg, immediately beneath the loaded area.

When a braking stress is applied, the two forces tending to shove the paving mixture ahead of the tire are:

(1) the horizontal braking stress fV acting on the contact area, bcih.

(2) the vertical load V from the pressure of the tire acting on the contact area bcih.

The influence of these two forces can be resolved into equivalent horizontal unit stresses acting toward the left in Figure 40, against the vertical face abcd of the wedge of pavement just ahead

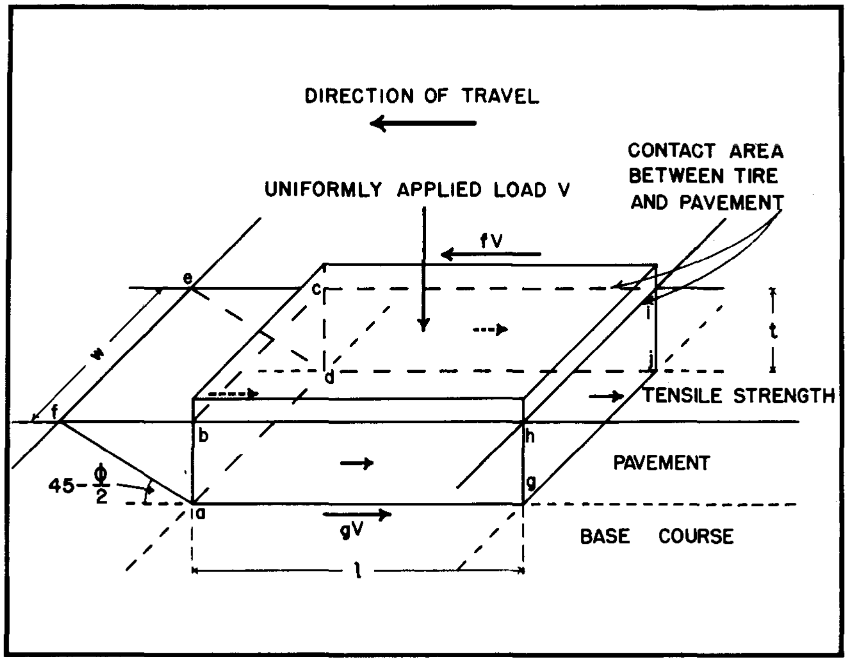



Figure 40. Illustrating the Forces Induced in a Bituminous Pavement by Braking Stresses

of the loaded area. The area of abcd is given by wt, where w is the width of the contact area, and t is the thickness of pavement.

The unit stress on the rectangular face abcd, corresponding to the braking stress on the contact area, is given by

$$\frac{(fV)\,(\ell w)}{wt} = \frac{fV\ell}{t} \tag{24}$$

Where f = coefficient of friction between pavement and tire. Its value may be as high as unity, but is usually below 0.8. The value of f tends to decrease as vehicle speed is increased[22].

V = average vertical pressure on the contact area.

$\ell$ = length of contact area.

w = width of contact area.

t = thickness of pavement.

However, the maximum braking stress, fV, cannot exceed the shearing resistance of the pavement under the loaded area,

$c + V \tan \phi$. Any tendency for fV to exceed $c + V \tan \phi$ would merely result in shearing of the pavement. This limitation on the maximum value of fV that can be developed, can be expressed by letting

$$\frac{fV}{c + V \tan \phi} = P \qquad \text{Where } P \leqq 1 \tag{9}$$

from which

$$fV = P\,(c + V \tan \phi) \tag{10}$$

Substituting this value for fV in equation (24) gives

$$\frac{fV\,\ell}{t} = \frac{P\,(c + V \tan \phi)\,\ell}{t} \tag{25}$$

The horizontal unit stress on the rectangular face abcd, due to the vertical pressure V of the tire on the contact area, is obtained by rearranging equation (3), giving,

$$L = V\,\frac{1 - \sin \phi}{1 + \sin \phi} - 2c\sqrt{\frac{1 - \sin \phi}{1 + \sin \phi}} \tag{26}$$

where the lateral pressure L is the unit stress required.

Consequently, the equivalent horizontal unit stress acting on the rectangular face abcd, due to the combined effect of the vertical load V and the braking force, fV, and tending to shove the pavement ahead of the loaded area (to the left in Figure 40), is given by the summation of the quantities on the right hand sides of equations (25) and (26), or

$$\frac{P\,(c + V \tan \phi)\,\ell}{t} + V\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) - 2c\sqrt{\frac{1 - \sin \phi}{1 + \sin \phi}} \tag{27}$$

The forces tending to resist the shoving of the pavement ahead of the tire by the vertical load and braking stress are,

(1) The frictional resistance gV, between pavement and base, acting over the rectangular surface adjg.
(2) The shearing resistance of the paving mixture along both sides, abhg and dcij, of the rectangular block.
(3) The resistance to displacement of the wedge of pavement abcdef immediately in front of the loaded area.
(4) The tensile strength of the pavement acting on the vertical face ghij at the rear of the rectangular block of pavement under the loaded area.

These four resisting forces can be resolved into equivalent horizontal unit stresses acting toward the right against the front end, abcd, of the rectangular block of pavement under the loaded area.

The horizontal reaction on the rectangular face abcd, equivalent to the frictional resistance between the pavement and base course, is given by

$$\frac{(gV)\ (\ell w)}{wt} = \frac{gV\ell}{t} \tag{28}$$

Where g = coefficient of friction between pavement and base course
and the other symbols have the significance previously defined for them.

The maximum frictional resistance between pavement and base, gV, however, cannot exceed the shearing resistance of the pavement under the loaded area, $c + V \tan \phi$. Any tendency for gV to exceed $c + V \tan \phi$ would merely result in shearing within the pavement itself. This limitation on the maximum value of gV that can be developed, can be expressed by letting

$$\frac{gV}{c + V \tan \phi} = Q \qquad \text{Where } Q \leqq 1 \tag{11}$$

from which

$$gV = Q\ (c + V \tan \phi) \tag{12}$$

Substituting this value for gV in equation (20) gives

$$\frac{gV\ell}{t} = \frac{Q\ (c + V \tan \phi)\ \ell}{t} \tag{29}$$

The shearing resistance of the paving mixture along the two vertical sides, abhg and dcij, can be obtained from the Coulomb equation $s = c + n \tan \phi$. The value of the normal pressure n in this case is given by equation (6), and is equal to $2cK\sqrt{\dfrac{1 + \sin \phi}{1 - \sin \phi}}$. Therefore, the horizontal unit reaction on the rectangular face abcd, equivalent to the shearing resistance of the paving mixture along the two vertical sides, abhg and dcij, of the rectangular block under the loaded area, is given by

$$\frac{2\left(c + 2cK\sqrt{\dfrac{1 + \sin \phi}{1 - \sin \phi}} \tan \phi\right) \ell t}{wt}$$

$$= 2\left(c + 2cK\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \tan \phi\right) \frac{\ell}{w} \tag{30}$$

The horizontal unit reaction on the rectangular face abcd equivalent to the maximum developed reaction of the wedge of pavement abcdef, immediately in front of the loaded area, is given by equation (8),

$$L = 2cJ\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} \tag{8}$$

The fourth source of pavement reaction to the braking stress listed was the pavement's tensile strength. The Mohr diagram indicates that this tensile strength should be $2c\sqrt{\frac{1 - \sin \phi}{1 + \sin \phi}}$. Because of the possible presence of fine hair cracks, etc., which might often prevent the development of the full tensile strength of the pavement, it seems desirable to neglect the tensile strength as a source of pavement reaction to braking stresses.

Therefore, the total horizontal unit reaction on the rectangular face abcd, Figure 40, equivalent to the four sources of pavement reaction to braking stresses listed, is given by the summation of the quantities on the right hand sides of equations (29), (30), and (8), or

$$\frac{Q\,(c + V \tan \phi)\,\ell}{t} + \frac{\left(2\quad c + 2cK\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\tan \phi\right)\ell}{w}$$

$$+ 2cJ\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} + \text{tensile strength} \tag{31}$$

To prevent pavement failure under the combination of vertical tire pressure V and braking stress fV, tending to shove the pavement ahead of the loaded area (to the left in Figure 40), the sum of the applied stresses must not exceed the sum of the reactions that can be developed by the pavement. This requirement for pavement stability is complied with when the sum of the effective applied stresses given by equation (27) is equated to the sum of the effective reactions developed by the pavement as indicated by equation (31). That is,

$$\frac{P\,(c + V \tan \phi)\,\ell}{t} + V\left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) - 2c\sqrt{\frac{1 - \sin \phi}{1 + \sin \phi}}$$

$$= \frac{Q\,(c + V \tan \phi)\,\ell}{w} + \frac{2\left(c + 2cK\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}}\tan \phi\right)\ell}{w}$$

$$+ 2cJ\sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} + \text{tensile strength} \tag{32}$$

Upon simplification, and neglecting the tensile strength, equation (32) becomes

$$V = c \left[ \frac{2\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\left(2K\frac{\ell}{w}\tan\phi + J + \frac{1-\sin\phi}{1+\sin\phi}\right) + 2\frac{\ell}{w} - \frac{\ell}{t}(P - Q)}{\frac{\ell}{t}(P - Q)\tan\phi + \frac{1-\sin\phi}{1+\sin\phi}} \right] \tag{33}$$

Equation (33) can be rewritten in terms of f - g, rather than P - Q, when it becomes,

$$V = 2c \left[ \frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\left(2K\frac{\ell}{w}\tan\phi + J\right) + \frac{\ell}{w} + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}}}{\frac{1-\sin\phi}{1+\sin\phi} + \frac{\ell}{t}(f - g)} \right] \tag{34}$$

It should be apparent that under braking and acceleration stresses, the frictional resistance between pavement and tire, fV, is opposed to the frictional resistance between pavement and base, gV. It is for this reason that the quantities P - Q and f - g appear in equations (33) and (34). Since neither P nor Q can be larger than unity, values for the term P - Q can vary only between -1 and +1.

Equations (33) and (34) are stability equations for the design of bituminous pavements subjected to braking or acceleration stresses. While at first glance they may appear to be rather formidable, they are made up entirely of quantities that are either provided by the conditions of design assumed for any given project, or that can be determined experimentally in the laboratory.

The length $\ell$ of the loaded area, and its width w, are established as soon as the wheel load and tire pressure are specified. The thickness of pavement t is usually designated more or less arbitrarily as 2 inches, 3 inches, etc. In equations (33) and (34), however, it should be observed that the quantities $\ell$, w, and t occur only as the ratios $\ell/w$ and $\ell/t$, which makes them easier to handle. An average value for $\ell/w$ for aeroplane and many truck tires is 1.5, with a maximum range of 1.0 to 2.0. The value of $\ell/t$, on the other hand, might vary more widely, depending upon the thickness of pavement and length of contact area. For pavements from 2 to 3 inches thick, the value of $\ell/t$ might vary from 3 to 6 for truck tires, and might be as high as 10 or more for the largest aeroplane tires. Values of c and $\phi$ for any proposed bituminous mixture are measured by the triaxial test. f is the coefficient of friction between pavement and tire and g is the coefficient of friction between pavement and base. P and Q are factors that are related to the maximum frictional resistance that

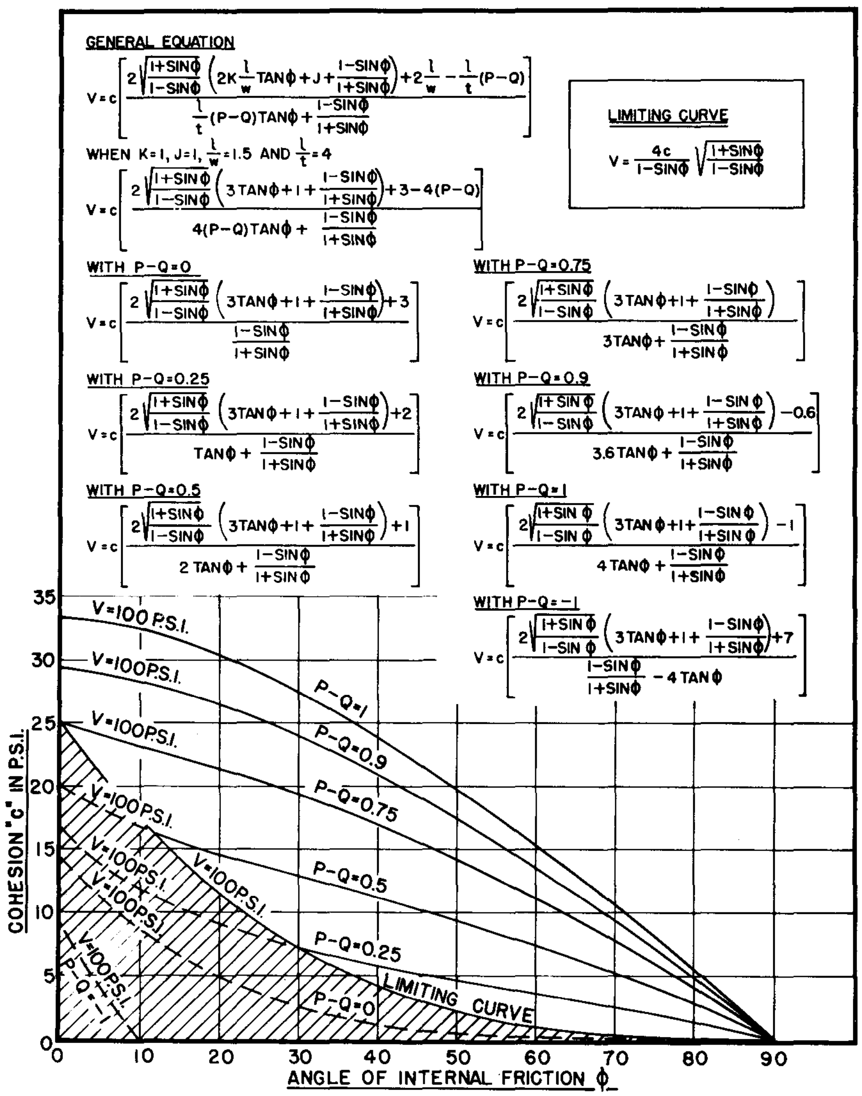



Figure 41. Illustrating Stability Curves for a Given Vertical Load for Bituminous Paving Mixtures Subject to Braking Stresses, in Terms of P - Q Values

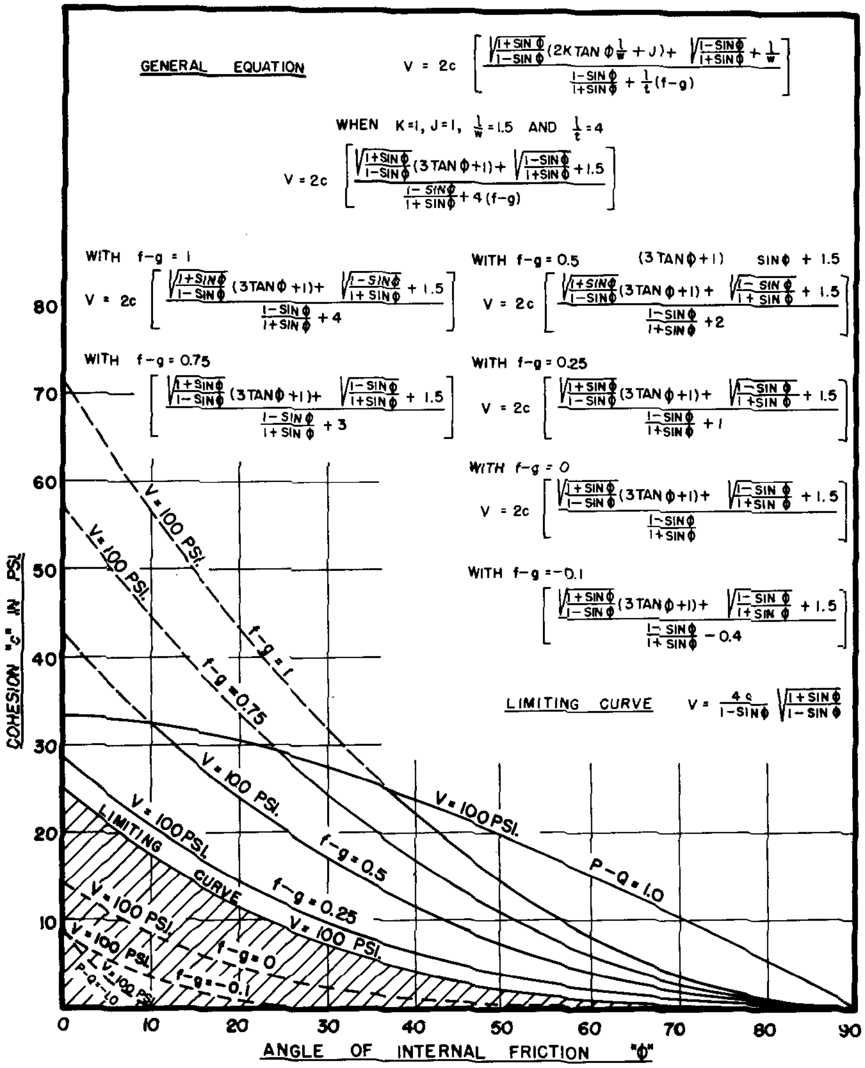



Figure 42. Illustrating Stability Curves for a Given Vertical Load for Bituminous Paving Mixtures Subject to Braking Stresses, in Terms of f - g Values

can be developed between pavement and tire, fV, and between pavement and base, gV, respectively. P and Q can be evaluated by field and laboratory tests and by utilizing equations (9) and (11), respectively. K and J are factors by which the unconfined compressive strength of the pavement must be multiplied to determine the lateral support L provided by the pavement adjacent to the loaded area in transverse and longitudinal directions, respectively. Representative values for K and J could be determined in the laboratory. In most cases, it would probably be quite conservative to assume that both K and J are equal to unity.

Figure 41 illustrates graphically the application of equation (33) to the design of bituminous paving mixtures that are to be subjected to braking stresses. Values of $K = 1$, $J = 1$, $\ell/w = 1.5$, and $\ell/t = 4$, have been assumed, and the stability curves, therefore, reflect the influence of different values of P - Q on the design of bituminous mixtures that must support a vertical load V of 100 p.s.i., in addition to being subject to braking stresses. It will be recalled from equations (9) and (11) that neither P nor Q can have a value greater than unity. Consequently, a value of $P - Q = 1$ means that the value of Q must be zero, and indicates that there is no frictional resistance between pavement and base. That this is the most critical condition of design for a pavement subject to braking stresses, is verified by the position of the curve labelled $P - Q = 1$ in Figure 41, which clearly requires bituminous paving mixtures with higher corresponding values of c and $\phi$ for a vertical load $V = 100$ p.s.i., than the stability curve for any other value of P - Q on the chart. The stability curves in Figure 41 for successively smaller values of P - Q indicate that bituminous mixtures with correspondingly smaller values of c and $\phi$ would be stable under braking stresses and a vertical load V of 100 p.s.i. The stability curve for $P - Q = 0$ represents the condition of design where the frictional resistance between pavement and base is equal to the braking stress between pavement and tire. The stability curve for $P - Q = -1$, is the opposite extreme of the case where $P - Q = 1$, and represents the highly undesirable design condition of no frictional resistance between pavement and tire (consequently, no actual braking stress), although there is full development of the frictional resistance between pavement and base.

Figure 42 is similar to Figure 41, but is a graphical illustration of equation (34), and demonstrates the nature of stability curves based directly upon f - g values. Figure 42 indicates the minimum values of c and $\phi$ required for pavement stability for different f - g values. Of particular interest are the dashed line portions of the stability curves for $f - g = 1$, $f - g = 0.75$, and

$f - g = 0.5$, in the middle left hand side of the diagram. The curve for $P - Q = 1$ represents the highest corresponding values that $f - g$ can have. Consequently, the dashed portions of the curves for $f - g = 1$, $f - g = 0.75$, and $f - g = 0.5$ indicate imaginary rather than real conditions of design. They are imaginary because they lie in a region of the diagram for which the value of P would have to be greater than unity. It was pointed out earlier in connection with equation (9) that P cannot have a value greater than unity, since this would mean that the frictional resistance between pavement and tire, fV, could be greater than the shearing resistance of the pavement itself. Figure 42, therefore, emphasizes the need for making use of the limiting P - Q curves for $P - Q = 1$ and $P - Q = -1$, to avoid the possibility of an unstable design that could result from employing f - g curves only.

In connection with the development of equations (33) and (34), it was pointed out that there are two forces which tend to cause the pavement to shove ahead of the tire. One of these forces is the braking stress of the tire on the pavement, and the other is the tendency of the vertical load V to squeeze the bituminous mixture out from under the loaded area. The resistances that are mobilized to oppose one of these forces do not necessarily oppose the other. The resistances opposing the braking stress are the four previously listed,

(a) Resistance of the wedge abcdef, Figure 40, to forward displacement
(b) Frictional resistance between pavement and base, on plane adjg, Figure 40.
(c) Shearing resistance of bituminous pavement along the two vertical faces, abhg and dcij, Figure 40.
(d) Tensile strength of the pavement acting on the face ghij, Figure 40.

However, when the sum of the second, third, and fourth of these resistances is just equal to the braking stress, only the first of these, that is, the resistance of the wedge abcdef, Figure 40, to forward displacement, can be mobilized to resist the tendency of the vertical load V to squeeze the pavement out from under the loaded area toward the front of the tire. The maximum horizontal resistance to forward displacement that can be mobilized by the wedge of pavement abcdef is given by equation (8),

$$L = 2cJ\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}} \qquad (8)$$

and the maximum vertical load V that can be applied to a bituminous pavement without causing it to be squeezed out in front of or behind the tire is obtained by substituting equation (8) in equation (3), which after simplification gives,

$$V = 2c\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}}\left(\frac{J\,(1 + \sin\phi) + (1 - \sin\phi)}{1 - \sin\phi}\right) \qquad (35)$$

When J = 1, equation (35) reduces to equation (5),

$$V = \frac{4c}{1 - \sin\phi}\sqrt{\frac{1 + \sin\phi}{1 - \sin\phi}} \qquad (5)$$

When applying equations (33) and (34) to the design of bituminous mixtures that will be exposed to braking stresses, therefore, it must be kept clearly in mind that the pavement must at the same time always have sufficient stability to resist being squeezed out either ahead of or behind the tire (or transversely), solely because of the vertical pressure V exerted by the tire on the loaded area. Consequently, there is a limit to the P - Q or f - g values, beyond which equations (33) and (34) are of little practical value for the design of bituminous pavements subject to braking stresses, because beyond this limit, the influence of the braking stresses provides a less critical criterion of design than the tendency of the vertical load V to squeeze the pavement out from under the tire.

As previously pointed out, when the sum of the second, third, and fourth of the resistances enumerated is just equal to the braking stress, equation (35) should be employed for design, to avoid squeezing out of the pavement from under the loaded tire in a longitudinal direction. Equation (35) becomes identical with equation (19) when f + g = 0 and K = J for the latter equation. It will be recalled that equation (19) was developed to prevent squeezing out of the pavement from under the loaded tire in a transverse direction under a stationary wheel load, or under a wheel load moving at uniform speed.

Negative values for f - g, Figure 42, indicate that the frictional resistance between pavement and base is greater than the frictional resistance between pavement and tire, and a positive value for $L_R$ should result, since at least a portion of the frictional resistance between pavement and base would be available to resist the tendency of the vertical load V to squeeze the pavement out from beneath the tire in a longitudinal direction. The limiting curves for this condition could be derived from equation (23), in which the value of f + g to be employed would be equal numerically to the negative value of f - g in Figure 42, or equation (34). That is, if f - g = -0.25, for example, the location of the corresponding limiting curve for Figure 42 would be obtained from equation (23) using a value for f + g = 0.25, and with the required value for J replacing that for K. In view of the subject matter of the previous section, it should also be observed that the shape of the curve of pressure distribution along the longitudinal axis of the contact area becomes important for determining the position of the limiting curve, when the values for f - g are negative. Consequently, for negative values of f - g, the corresponding limiting curves would be below that shown in Figure 42, which is, therefore, conservative.

It should be clear that stability diagrams similar to Figures 41 and 42, based on equations (33) and (34), respectively, for other combinations of values for J, K, $\ell$/w, and $\ell$/t, could be easily prepared. It should also be noted that for the particular values for J, K, $\ell$/w, and $\ell$/t employed for Figures 41 and 42, corresponding stability curves for any other value of vertical load V can be obtained on the basis of simple proportionality. Suppose for example, that the corresponding curve for P - Q = 1 in Figure 41 was required for V = 50 p.s.i. For all values of $\phi$ along the abscissa, points marking off one-half the ordinate between the abscissa and the curve labelled P - Q = 1 for V = 100 p.s.i. would be shown. The curve through these points would be required curve for P - Q = 1, when V = 50 p.s.i. Similarly, if for all values of $\phi$ along the abscissa, points marking off twice the value of the ordinate between the abscissa and the curve labelled P - Q = 1 for V = 100 p.s.i. are drawn, the curve through these points will represent the curve for P - Q = 1 when V = 200 p.s.i. This procedure of simple proportionality for obtaining stability curves for other values of V than those for V = 100 p.s.i., shown in Figure 41, is possible because it will be observed from equation (33) that cohesion c is a multiplier for the balance of the quantity on the right hand side of the equation. Consequently, if every factor on the right hand side is constant except c, the value of V depends directly on the value of c, and vice versa.

A similar procedure can be employed to obtain additional stability curves for most of the other illustrative stability diagrams contained in this paper, e.g. Figures 14, 17, and 42. It will be noted that like equation (33), the stability equations on which each of these stability diagrams are based, have cohesion c as a multiplier for the remainder of the equation on the right hand side.

It will be observed that equations (33) and (34) contain the variable t, representing pavement thickness. Consequently, the influence of changes in pavement thickness on the stability of bituminous pavements subjected to braking or acceleration stresses can be investigated.

For positive values of f - g, this relationship between pavement thickness and pavement stability is illustrated indirectly in Figure 43, in which the influence of changes in the ratio of $\ell$/t on the design of a bituminous pavement subject to braking stresses is indicated. The symbol $\ell$ refers to the length of the contact area, Figure 40, while t represents pavement thickness. As in similar diagrams in this paper, each $\ell$/t curve in Figure 43 indicates the minimum corresponding values for c and $\phi$ required for bituminous mixtures that are to be stable for the design conditions represented by that curve. It is clear from Figure 43, that for positive values of f - g, and with all other factors equal, the higher the value of the ratio $\ell$/t, the more stable must be the paving mixture (higher corresponding values for c and $\phi$), when the pavement is subjected to severe braking stresses under a given vertical load V. Higher values for the ratio $\ell$/t result either when L is kept constant and the pavement thickness t is decreased, or when t is maintained constant and the length $\ell$ of the contact area is increased.

Figure 43 demonstrates that pavement thickness is an asset for pavements that are to be subjected to severe braking stresses, when f - g is positive. That is, to resist a given braking stress, a thin pavement must be designed to have a higher minimum stability (higher corresponding values for c and $\phi$) than a thick pavement. A study of Figure 40 demonstrates that this latter conclusion is to be expected, since the resistance to braking stresses increases with an increase in the shearing area of the sides abhg and dcij. For a given length $\ell$ of contact area, it is apparent that the area of these two sides will increase when t is increased (that is, as the ratio $\ell$/t is decreased), and vice versa.

It is to be particularly noted that the conclusions expressed above in connection with Figure 43 apply to the design of bituminous pavements that are to be subjected to severe braking stresses, only when the values for f - g in equation (34) are positive.

An examination of equation (34) indicates that when f - g = 0, that is when the frictional resistance between pavement and tire generated by the braking stress is equal to the maximum frictional resistance between pavement and base, pavement thickness has no influence on the stability developed by the pavement under braking stresses.

A study of equation (34) also shows that for negative values of f - g, that is, when the frictional resistance between pavement and

GENERAL EQUATION

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\ 2K\tan\phi\,\frac{1}{w} + J + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + \frac{1}{w}}{\frac{1-\sin\phi}{1+\sin\phi} + \frac{1}{t}(f-g)}\right]$$

WHEN $f-g = 0.25$, $K=1$, $J=1$ AND $\frac{1}{w} = 1.5$, THE GENERAL EQUATION BECOMES

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi} + 0.25\,\frac{1}{t}}\right]$$

WHEN $\frac{1}{t} = 0$

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi}}\right]$$

WHEN $\frac{1}{t} = 0.5$

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi} + 0.125}\right]$$

WHEN $\frac{1}{t} = 1$

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi} + 0.25}\right]$$

WHEN $\frac{1}{t} = 2$

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi} + 0.5}\right]$$

WHEN $\frac{1}{t} = 4$

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi} + 1}\right]$$

WHEN $\frac{1}{t} = 10$

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi} + 2.5}\right]$$

WHEN $\frac{1}{t} = 20$

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi} + 5}\right]$$

WHEN $\frac{1}{t} = 50$

$$V = 2c\left[\frac{\sqrt{\frac{1+\sin\phi}{1-\sin\phi}}\,(3\tan\phi+1) + \sqrt{\frac{1-\sin\phi}{1+\sin\phi}} + 1.5}{\frac{1-\sin\phi}{1+\sin\phi} + 12.5}\right]$$

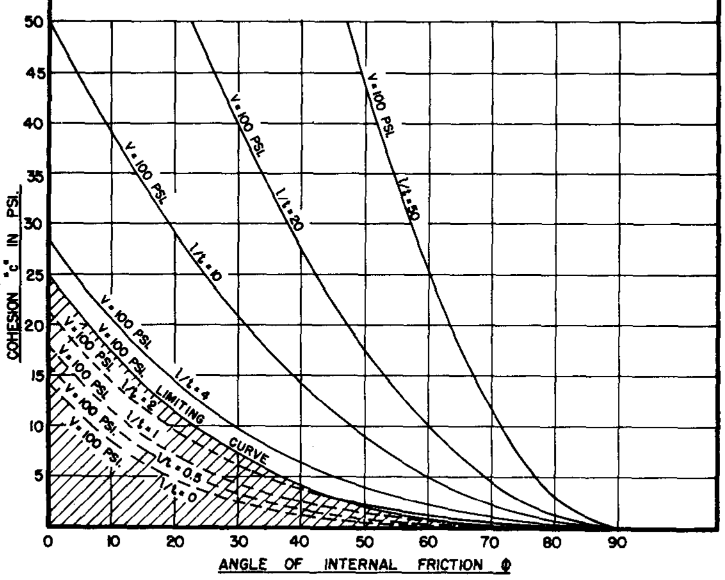



**Figure 43. Influence of Thickness on the Design of Bituminous Pavements Subject to Severe Braking Stresses, for Values of f - g.**

tire generated by the braking stress is less than the maximum frictional resistance between pavement and base, an increase in pavement thickness leads to a decrease in the stability developed by the pavement under braking stresses.

Consequently, equation (34) indicates that the influence of pavement thickness on the stability of a pavement subject to braking stresses depends upon the value of f - g involved. Pavement stability increases with pavement thickness for positive values of f - g; pavement thickness has no influence on pavement stability when f - g = 0; and pavement stability decreases with an increase in pavement thickness when f - g has negative values.

However, as pointed out earlier in this section, when f - g = 0, or has negative values, the tendency of the vertical load V to squeeze the pavement out from under the tire seems to present a more critical condition of design than the braking stress itself.

## DISCUSSION OF STABILITY CRITERIA FOR BITUMINOUS PAVEMENT DESIGN

In the introduction it was pointed out that there are three principal conditions of pavement stability to be considered when designing bituminous paving mixtures.

(a) Stability under stationary wheel loads.
(b) Stability under wheel loads moving at a relatively high but uniform rate of speed.
(c) Stability under the braking and accelerating stresses of traffic.

Practical field observation has indicated that pavement instability can usually be classified in one or more of the following three different types,

(a) Squeezing out of the pavement from under the wheel in a transverse direction for either stationary and moving loads,
(b) Shoving of the pavement at bus stops, traffic lights, etc., probably caused by braking and acceleration stresses, and the development of washboard occasionally in unstable pavements carrying moving traffic,
(c) Actual tearing of the pavement under moving traffic, generally when a relatively thin layer of pavement has been placed on a base to which it is either poorly bonded or not at all.

The design requirements, in terms of minimum values for cohesion c and angle of internal friction $\phi$, for stable bituminous pavements for all three types of loading, stationary wheel loads, wheel loads moving at a uniform rate of speed, and wheel loads that exert severe braking and acceleration stresses, have been shown to depend upon,

(1) lateral support L provided by the pavement adjacent to the loaded area;
(2) pavement thickness;
(3) shape of the curve of pressure distribution on the contact area;
(4) rate of loading or rate of strain;
(5) frictional resistance between pavement and tire;
(6) frictional resistance between pavement and base;
(7) stability in the direction of the longitudinal versus the transverse axis of the tire contact area.

Other factors than the seven just enumerated contribute to pavement stability or instability of course, one of the most obvious being density, and another temperature. However, this paper has been primarily interested in attempting to describe the role that each of the above seven items seems to play in establishing pavement stability, and for this purpose it has been assumed that 'all other factors are equal'.

The amount of lateral support L provided by the portion of any given pavement adjacent to the loaded area can probably be most effectively influenced in the design and construction stages by requiring adequate consolidation of the pavement. Experimental work is required to measure the actual magnitude of the lateral support L provided by the pavement adjacent to the loaded area, for different pavement designs. This may consist of measuring the value of the factor K, Figure 17, or may involve an entirely different approach for determining the value of the lateral support L.

Pavement thickness is usually specified more or less arbitrarily and for bituminous mixtures is normally from 2 to 4 inches. It has been demonstrated that for stationary loads, and for wheel loads moving at a relatively uniform rate of speed, a thick pavement requires greater stability (higher corresponding minimum values for c and $\phi$) than a thin pavement, Figures 32, 37, and 38. On the other hand, for pavements subject to severe braking or acceleration stresses, equation (34) shows that pavement stability increases with pavement thickness for positive values of $f - g$, Figure 43; pavement thickness has no influence on pavement stability when $f - g = 0$; and pavement stability decreases with an increase in pavement thickness when $f - g$ has negative values. However, when designing for braking stresses, it appears that when $f - g = 0$, or has negative values, the tendency of the vertical load V to squeeze the pavement out from under the tire seems to present a more critical condition of design that the braking stress itself. These facts should be kept carefully in mind.

Figure 39 has demonstrated the influence that changes in the shape of the curve of pressure distribution on the contact area can have on bituminous pavement design. Any marked change in the shape of this pressure distribution curve would require modifications in the design of the tires that transmit the wheel load to the pavement. More information is required concerning the exact shape of the curve of pressure distribution on the contact area, for the wide range of tires for automobile, truck, aeroplane, tractor, etc., in current use, and concerning the influence of tire loading and inflation pressure on the shape of the pressure distribution curve for each type of tire.

The influence of the rate of loading, or rate of strain, on the value of cohesion c, and possibly on the value of $\phi$, has been demonstrated in Figures 18(b) and 19. Equation (3) indicates that an increase in c increases the stability that any bituminous mixture can develop, the increase in stability being directly proportional to the increase in c. After more information on this variable has been obtained and correlated with field performance, it should be possible to take advantage of the influence of rate of loading, or rate of strain, on pavement stability, when establishing the minimum design requirements in terms of c and $\phi$ for stationary loads versus loads moving at a uniform rate of speed. Experienced bituminous engineers have observed pavements that are stable under moving vehicles, but into which the wheels of the same vehicles would settle rapidly if they stopped. This is also sometimes observed on freshly constructed bituminous pavements. Present evidence indicates, therefore, that paving mixtures with much smaller corresponding values for c and $\phi$ could be selected for stable pavements for wheel loads moving at a reasonably uniform rate of speed, than for stationary loads. However, this difference in design could not be considered unless there was reasonable certainty that throughout its life the pavement would be subjected only to wheel loads moving at a relatively uniform speed. Bituminous pavements for highways and roads in many rural areas would be possible examples.

Frictional resistance between pavement and tire and between pavement and base appears to have a remarkable influence on pavement stability under all three types of loading, stationary, moving at a uniform rate of speed, and subject to braking or acceleration stresses. High frictional resistance between pavement and tire is desirable and necessary for safe driving. Excellent work on evaluating this factor has already been done by Moyer[22], and by Giles and Lee[23]. However, further research is required to develop bituminous mixtures that are satisfactory in every other respect and that will provide and maintain throughout their life a high coefficient of friction f between pavement and tire. Work is also required to measure the changes in the value of f that are likely to occur with changes in the surface of a pavement with time and under various weather and service conditions. Figures 27 and 28 show that low values of $P + Q$ or $f + g$ provide the most critical conditions of design for pavements for stationary loads and for wheel loads moving at a relatively uniform rate of speed. Figures 41 and 42 indicate that high values for $P - Q$ or $f - g$ provide the most critical conditions of design for pavements subjected to braking or acceleration stresses. Consequently, pavement design should apparently be based upon the minimum value of f expected for the pavement throughout its life, for stationary loads, and for wheel loads moving at a relatively uniform rate of speed, and upon the maximum value of f, expected for the pavement in its lifetime, for braking and acceleration stresses, since this leads to the selection of the minimum values of c and $\phi$ required to provide stable pavements under each of these three conditions of loading. The influence on the frictional resistance between pavement and tire of the various seal coats that may be applied throughout the life of a pavement must also be considered in pavement design.

No work seems to have been done so far towards measuring the coefficient of friction, g, between bituminous pavements and the various types of base courses on which they are laid. Research for the purpose of evaluating this variable under many different conditions is highly desirable, because of its importance in the rational design of bituminous pavements. As with the variable f, it is possible that the value of coefficient of friction g between any given pavement and base course may change with time, due to such factors as degradation of the aggregate, infiltration of water, stripping of the bitumen from the aggregate in the vicinity of the interface between pavement and base, and the loosening or strengthening of the bond between pavement and base for any reason with time.

It very often happens that a bituminous pavement will be exposed to two or to all three of the different types of loading, stationary, moving at a relatively uniform rate of speed, and subject to braking and acceleration stresses. Stationary loads, and wheel loads moving at a reasonably uniform rate of speed, tend to squeeze out the pavement from under the wheel in the transverse direction of the loaded area, while braking and acceleration stresses tend to shove the pavement in the longitudinal direction of the tire contact area. Consequently, pavement design should be checked for the wheel load stresses that are anticipated in the directions of both the longitudinal and transverse axes of the tire contact area and the most critical of these should provide the actual basis for design.

In addition to the above brief discussion of the effect on pavement stability of each of the seven factors listed, it is worth while to consider further the influence of items (5) and (6), the frictional resistance between pavement and tire and between pavement and base, on the design of bituminous pavements for stationary wheel loads, for wheel loads moving at a relatively uniform rate of speed, and for loads that subject pavements to severe braking stresses.

The effect of frictional resistance between pavement and tire and between pavement and base on the design of bituminous mixtures subjected to stationary loads, and to wheel loads moving at relatively uniform speed, is demonstrated in Figures 27 and 28, for a vertical load $V = 100$ p.s.i. Low minimum values for c and $\phi$ for satisfactory pavement design result from high values for $P + Q$, or $f + g$. High values for $P + Q$ or $f + g$ are obtained when the frictional resistance between pavement and tire and between pavement and base are both high.

When designing pavements subject to braking stresses on the other hand, Figures 41 and 42 demonstrate the advantage of keeping the values of $P - Q$ or $f - g$ as low as possible, since this permits the use of paving mixtures with low values of c and $\phi$. For safety reasons, it is desirable to design and construct bituminous pavements having a high coefficient of friction between pavement and tire. This in turn tends to provide high values for P or f. Therefore, if the value of $P - Q$ or $f - g$ is to be kept as low as possible, the value of Q or g, that is, the frictional resistance between pavement and base, must also be kept as high as possible.

It is clear, therefore, that a high value for frictional resistance between pavement and base is to be greatly desired when designing bituminous paving mixtures for all three conditions of loading, stationary wheel loads, wheel loads moving at a uniform speed, and severe braking stresses. Expressed in another way, there should always be a strong bond between pavement and base, because it contributes materially to pavement stability under each of the three different types of wheel loads.

However, insofar as braking stresses are concerned, if the bond between pavement and base is strong and approaches the shearing resistance of the paving mixture, resulting in a low value for P - Q or f - g, the limiting curve in Figures 41 and 42 indicates that the tendency of the vertical load V to squeeze the pavement out from under the wheel may be a more critical condition of design than the action of the braking stresses themselves.

When P - Q or f - g is positive, equations (33) and (34), respectively, indicate the importance of both pavement thickness and adequate bond between pavement and base, when designing for pavement stability to resist braking stresses. When resurfacing over an old pavement, or when building a new pavement over a smooth dense base, the bond between old and new pavement, or between pavement and base, may be rather weak unless adequate care is taken. This may result in a high value for P - Q or f - g. If the old pavement or base is uneven, the new pavement may be relatively thin in some areas. Thin sections may also occur during the spreading operation when road-mixing. When f - g is positive, it is apparent from equation (34) that for a bituminous pavement with given values for c and $\phi$, any factors that increase the values of either $\ell/t$ or f - g, or both, will decrease the stability V of the pavement under braking or acceleration stresses. The length $\ell$ of the contact area is constant for any given wheel load and tire pressure. Therefore, $\ell/t$ will increase as the thickness t of the pavement is decreased. The value of f - g tends to increase as the value of g is decreased, that is as the bond between pavement and base becomes weaker. Consequently, for positive values of f - g, equation (34) demonstrates that the stability of a bituminous pavement under braking and acceleration stresses is improved by adequate pavement thickness, and by taking proper measures to obtain a strong bond between pavement and base. On the other hand, as previously pointed out, for pavements subject to severe braking or acceleration stresses, equation (34) also indicates that pavement thickness has no influence on pavement stability when f - g = 0, and pavement stability decreases with an increase in pavement thickness when f - g has negative values. However, when designing for severe braking stresses, it appears that when f - g = 0, or has negative values, the tendency of the vertical load V to squeeze the pavement out from under the tire seems to present a more critical condition of design than the braking stress itself.

For the case of pavement stability at bus stops, traffic lights, and other areas where vehicles may come to a full stop, the question arises as to whether the pavement for these sections should be designed for moving or for stationary loads. During most of the period when the brakes are bringing the vehicles to a full stop, the value of cohesion c developed in the pavement may be much greater than its magnitude under a stationary load. On the other hand, for a very brief interval just before the vehicle stops, the rate of movement becomes so low that the value of cohesion c developed may not be much larger than for a stationary load. Consequently, if there is a weak bond between pavement and base at an area subject to much stopping and starting of traffic, the design curve indicated by one of the appropriate larger values of f - g in Figure 42 would be selected as the basis for design, using a low rate of strain when testing the bituminous mixture in the triaxial test. However, if an excellent bond between pavement and base is assured, resulting in a low f - g value, pavement design for bus stops, traffic lights, and other areas where there is much starting and stopping of traffic, should probably be based upon preventing the squeezing out of the pavement from under the tire.

For pavements subject to traffic moving at relatively high rates of speed, and where, although braking stresses may be applied, they are used only to obtain temporary deceleration and do not bring vehicles to a full stop, e.g. normal highway traffic in rural areas and the runways at airports, it would seem reasonable to base bituminous mixture design on moving loads. That is, a relatively high rate of strain would be used when testing the paving mixture in triaxial equipment, in order that a value of cohesion c for the mixture might be obtained in the laboratory for design purposes, that would approximate the value of cohesion c developed in the resulting pavement under moving traffic. Further experimental work is required to determine what the permissible rate of strain should be for the triaxial testing of bituminous mixtures that are to carry moving wheel loads.

For pavements subject to stationary wheel loads, or to very slowly moving traffic, bituminous mixture design should probably be based upon static loads. The pavements for most city streets, all parking areas, and the aprons, taxiways, and turnaround areas at the ends of runways at airports, are representative examples. A very low rate of strain should be used for triaxial tests on bituminous mixtures that are proposed for pavements for these areas.

Finally, until there has been an opportunity to build up information on the field performance of bituminous mixtures designed by the rational method and employing the triaxial test, it would seem prudent to base the design of bituminous mixtures on the stationary load condition, for the maximum pressure applied to the contact area[15,16], and possibly including an impact factor.[24] If particular care is taken to obtain a strong bond between pavement and base, it is believed that the curves of Figure 14, for which K = 1, would provide a conservative basis of design. Later, as confidence in this method of design becomes established, and as more experimental data becomes available concerning average values and the range of values possible for each of the important variables, the refinements indicated in the paper, particularly with regard to Figures 17, 27, 28, 30, 35, 36, 37, 39, 41, 42, and 43, which might lead to a less conservative design for both static and moving loads, could be gradually adopted.

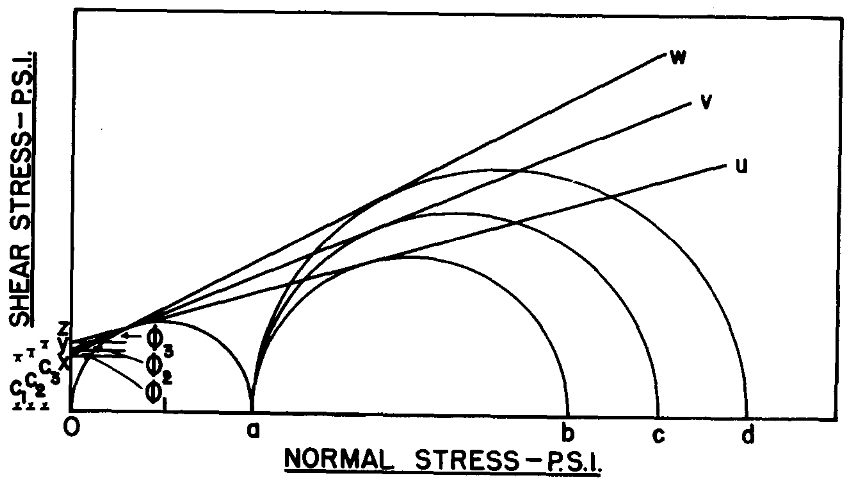
## RELATIONSHIPS BETWEEN UNCONFINED COMPRESSION AND TRIAXIAL TESTS

Several different forms of the unconfined compression test have been proposed from time to time as methods for measuring the stability of bituminous mixtures. The most recent development of this test to obtain some prominence is the Marshall stability test.

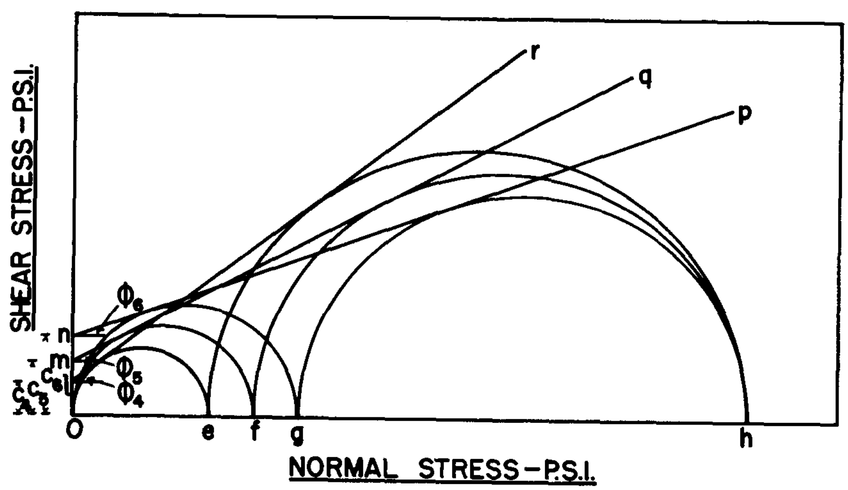
It has been the principal purpose of this paper to indicate how the triaxial test can be employed to measure the strengths of bituminous paving mixtures under conditions of stress similar to those to which they are subjected by traffic in the field. However, this test is rather time consuming, and it would be highly advantageous if some simpler and more rapidly performed test could be correlated with the triaxial test and used for the checking and control of the stability of bituminous mixtures in the field. Since the unconfined compression test appears to be of the same general type, it is worth while to determine whether there is any useful relationship for this purpose. Figure 44 illustrates the discussion of this point, which demonstrates that the unconfined compression test can be quite misleading, insofar as indicating the stability that bituminous paving mixtures can develop in the field is concerned.

In Figure 44(a) the Mohr envelopes xw, yv, and zu represent three different bituminous mixtures with corresponding values for cohesion c and angle of internal friction $\phi$, $c_1\phi_1$, $c_2\phi_2$, and $c_3\phi_3$, respectively. Similarly, in Figure 44(b) the Mohr envelopes lr, mq, and np represent bituminous mixtures with corresponding values for cohesion c and angle of internal friction $\phi$, $c_4\phi_4$, $c_5\phi_5$, and $c_6\phi_6$, respectively.

Figure 44(a) indicates that the unconfined compressive strength 0a is exactly the same for each of the quite different bituminous




(a)




(b)

Figure 44. Demonstrating the Inadequacy of an Unconfined Compression Test for Measuring the Stability of Bituminous Paving Mixtures

mixtures represented by Mohr envelopes xw, yv, and zu. For an unconfined compressive strength test, the amoung of lateral support provided is zero. On the other hand, bituminous pavements in place in the field are able to mobilze lateral support from the pavement adjacent to the loaded area. In an earlier part of this paper, it was indicated that the unconfined compressive strength was a measure of this lateral support. Consequently, the stability under service conditions of the three bituminous mixtures represented by Mohr envelopes zw, yv, and zu may be indicated by Mohr circles ad, ac, and ab, respectively. If traffic loads exert a vertical pressure V equal to point d in Figure 44(a), then only the bituminous mixture represented by the Mohr envelope xw would be stable in the field, and the other two represented by Mohr envelopes yv and zu would be unstable. It should be noted again that the unconfined compression test would give all three of these bituminous mixtures exactly the same stability rating, 0a. Consequently, Figure 44(a) demonstrates that the unconfined compression test may indicate exactly the same stability for bituminous mixtures that would have widely different stabilities under field conditions.

Figure 44(b) represents the reverse of the situation illustrated in Figure 44(a). The unconfined compressive strength of each of the three bituminous mixtures lr, mq, and np is indicated by 0e, 0f, and 0g, respectively. Since the unconfined compressive strength is a measure of the lateral support provided by the pavement adjacent to the loaded area, Figure 44(b) demonstrates that these three bituminous mixtures are all capable of developing exactly the same resistance 0h to the applied vertical load, under the conditions that exist in the field. Nevertheless, if the stability of these three mixtures were evaluated by the unconfined compression test, three widely different stability ratings 0e, 0f, and 0g, would be indicated. If the vertical load V to be carried is equal to 0h, the triaxial test would indicate that the three bituminous mixtures had the same stability, and that all three would be satisfactory. If the minimum stability requirement according to the unconfined compression test were given by 0g, then this test would indicate that only the bituminous mixture represented by Mohr envelope np would have the necessary stability under field conditions and that the mixtures represented by Mohr envelopes lr and mq would be unstable. Therefore, Figure 44(b) shows that the unconfined compression test could reject as being unstable, bituminous paving mixtures that would develop quite adequate stability under field conditions.

Figure 44 clearly demonstrates, therefore, that the unconfined compression test, and probably all similar tests such as the Marshall test, that are closely related to it, are fundamentally incapable of providing trustworthy measurements of the stability that bituminous paving mixtures can develop under field conditions. Figure 44(a) shows that the unconfined compression test is capable of indicating adequate stabilities for bituminous mixtures that would be unstable in the field. Figure 44(b) demonstrates that the unconfined compression test is also capable of rejecting bituminous mixtures that would have adequate stability under field conditions.

It is sometimes stated in defense of unconfined compression and other similar empirical stability tests that they are capable of reflecting changes in aggregate gradition, filler content, bitumen content, etc. Figure 44(a) demonstrates that such changes could be made in the composition of bituminous mixtures which would have little or no effect on their unconfined compressive strength, but that would at the same time greatly influence the stability of the pavement under the conditions of stress to which it is exposed in the field. On the other hand, Figure 44(b) indicates that wide changes can be made in the composition of bituminous mixtures, which would cause large variations in their unconfined compressive strength values, but that would have little influence on pavement stability under field conditions. Consequently, it is apparent from Figure 44 that the unconfined compression test either may or may not be influenced by wide changes in the composition of bituminous mixtures, which are reflected by large variations in their c and $\phi$ values. Furthermore, the influence on the unconfined compression test values caused by these changes in composition may lead to quite erroneous conclusions concerning the influence of these changes on pavement stability under field conditions.

There seems to be little doubt that engineers can be led seriously astray when endeavouring to measure the stability of bituminous mixtures by the unconfined compression test, by any of the variations of this test, such as the Marshall stability test and probably the Hubbard Field stability test, or by any other stability test that can be correlated with the unconfined compression test. Figure 44 indicates that these tests may actually be so misleading that it may be questioned whether they serve any useful purpose as stability tests. The other empirical stability tests in common use at this time should also be suspect, unless it can be shown that they meet the fundamental requirements that any satisfactory stability test for bituminous paving mixtures must possess.

At the present time, we know of no test that can provide a satisfactory measure of the stability of bituminous pavements, except the triaxial test. While it is rather complex and time consuming as currently performed, it seems to be capable of furnishing data from which the actual stability of bituminous mixtures under field conditions can be calculated, and on which a rational method of design for the strength of bituminous pavements can be based. Further research may develop a simple rapidly performed test, useful for field control, that can be closely correlated with the triaxial test.

## GENERAL

A few general comments will be made in this section that could not be included elsewhere in this paper.

1. The mathematical development, that appears to be required for a rational method of design for bituminous paving mixtures based upon the triaxial test, may be considered by some engineers to be a serious barrier to the routine use of this method. A similar reaction was probably occasioned by the publication of the Westergaard equations for a rational method of design for rigid pavements about twenty years ago. However, the Westergaard equations can be plotted in the form of a simple graph, from which the required thickness of slab can be obtained from the appropriate wheel load curve in a few seconds, if the value of subgrade modulus, k, and flexural strength of the concrete to be employed are known. Consequently, practicing engineers do not normally make direct use of the Westergaard equations, but employ the relatively simple graph or graphs that have been prepared from these equations.

As the many illustrative diagrams throughout the present paper indicate, the mathematical equations required for a rational method of design for bituminous pavements can also be expressed in the form of simple graphs. As soon as general agreement can be reached with regard to the mathematical equations to be employed, and pertaining to the rate of strain at which the triaxial test is to be run to provide representative values for c and $\phi$ for the bituminous mixture proposed for any given project, the mathematical development could be disregarded, and only the appropriate design charts based upon the various equations employed. The values for c and $\phi$ obtained from the triaxial test provide the coordinates of a point on the appropriate chart. The position of this point relative to the particular stability curve specified by the design requirements for the project would indicate whether or not the proposed bituminous pavement had the required stability.

2. Triaxial tests must be made on briquettes of bituminous mixtures formed in the laboratory. It is highly important that the bituminous mixture in the compacted briquette for the triaxial test should duplicate as nearly as possible the structure that the same bituminous mixture will attain in the road surface. Endersby[1] and Hveem[6] have provided a very fine account of the work that has been done, and that is still going on, under the auspices of The Triaxial Institute, to develop a laboratory compaction device that will provide samples for the triaxial test, having a structure identical with that developed under rolling and traffic in the field. The development of such a compaction test is of the greatest importance, since stability measurements on a laboratory sample are of questionable value, unless the laboratory sample duplicates the characteristics and structure that would be developed in the field by the bituminous mixture being tested.

It is not improbable that the structure of a bituminous pavement is somewhat different in a horizontal than in a vertical direction. If this should be the case and if it is of sufficient importance, the value of the unconfined compressive strength to be employed for evaluating the lateral support L provided by the pavement adjacent to the loaded area might have to be measured by the procedure employed by Haefeli at the University of Zurich in Switzerland for running the triaxial test, in which the vertical load V becomes the minor principal stress, and the horizontal pressure L is the major principal stress.

3. The comprehensive investigation of the design of bituminous mixtures conducted by the U. S. Corps of Engineers[19] has provided some excellent quantitative data on the increase in the density of bituminous pavements that occurs under traffic. Their studies have shown that the air void content of what might ordinarily be considered to be well designed bituminous mixtures, may approach zero with the increased density resulting from traffic, and serious loss of stability may occur.

In the triaxial testing of a bituminous mixture, therefore, it would seem advisable to determine its stability at the density the pavement will have when rolling is complete, and also at the ultimate density it may be expected to eventually acquire under traffic, since both conditions may be critical in the life of the pavement.

4. The values for c and $\emptyset$ determined for a bituminous paving mixture by the triaxial test depend upon the temperature at which this test is made. This test temperature should be correlated with the most critical temperature anticipated for the bituminous

pavement in service, that is, the temperature at which the pavement will have the lowest stability. In North America, the critical pavement temperature is usually considered to be 140° F. Consequently, to provide the minimum stability required by pavements at 140° F., values for c and $\phi$ for paving mixtures should be determined by a triaxial test performed at this temperature.

5. The dimensions of the specimen tested in triaxial compression must receive careful consideration, if representative values for cohesion c and angle of internal friction $\phi$ are to be obtained.

To avoid the effects of friction between the end plates and the test briquette during loading, and the direct transfer of load between the two plates, the height of the specimen should be at least twice its diameter. If the angle of internal friction $\phi$ is likely to approach 40 to 45°, the height should be at least two and one half times the diameter. Nijboer[2] recommends a height to diameter ratio of three for asphaltic concrete mixtures, and a ratio of two and one half for sand asphalt, including sheet asphalt mixtures.

The diameter of the test briquette should be not less than four times the diameter of the largest particles in the bituminous mixture. Smith[3] reports excellent reproducibility of test results for this ratio. Nijboer,[2] on the other hand, prefers that the diameter of the test specimen should be at least six times the diameter of the largest particles. He points out that in this case the cross-sectional area of a single large particle is only about 3 per cent of the cross-sectional area of the test briquette.

6. In the development of the rational method of design for bituminous paving mixtures outlined in this paper, no mention has been made of the influence of the repeated loadings of highway or airport traffic on design.

This is a matter that may have to be answered by practical observations of the field performance of bituminous mixtures designed by this method.

On the other hand, the Corps of Engineers' investigation[19] indicated that one of the effects of continued traffic on bituminous pavements was a very marked increase in density. As long as this density increase does not eventually reduce the air voids below the critical value, normally about 2 to 3 per cent by volume, at which stability may begin to decrease, the influence of repeated traffic loads should ordinarily be beneficial, and should provide an increase in stability rather than otherwise. Consequently, it may be that as long as continued traffic does not increase the pavement density to the point where the volume of air voids becomes critically low, no provision in design for the repeated loads of traffic may be necessary. In addition, it is well known that bituminous binders tend to harden with time, and to develop an internal structure of their own. This gradual hardening of the binder should increase the stability of bituminous pavements with time and may be adequate in itself to compensate for the effect of repeated traffic loads on pavement stability, if this is an item that must be taken into account.

In view of these considerations, it may not be unreasonable to ask whether a bituminous pavement, that eventually shows evidence of instability under the repeated traffic of vehicles with some maximum wheel load and tire pressure, was actually designed for that magnitude of load in the first place. It may have been underdesigned from the beginning.

Nevertheless, if, in spite of these observations, field experience should indicate that repetitions of traffic loads must be considered in connection with the stability of bituminous pavements, they could probably be satisfactorily taken into account by multiplying the design load by a traffic factor. This would be equivalent to employing a safety factor.

7. Not enough is known about the actual magnitude of the variables K, f, g, P, Q, n, t, etc., and the shape of the curve of pressure distribution on the contact area, to make adequate use of them for the design of bituminous pavements at the present time. Considerable laboratory and field investigation is needed to provide average values and the range of values possible for these different factors. The development of reliable testing procedures will be required in the case of some of these variables, in order that they may be quickly evaluated for individual bituminous mixtures, or for the paving projects on which any given paving mixture is to be laid.

8. Good correlation between method of design and field performance is sometimes reported for bituminous pavements designed by the three empirical tests in common use, Hubbard-Field, Marshall, and Hveem Stabilometer. Nevertheless, one is inclined to enquire just how carefully this correlation has been established, and what price in overdesign has frequently been paid where it is thought to be demonstrated. It is a serious disadvantage of the use of these empirical tests that the actual degree of either overdesign or underdesign for a bituminous pavement is never known. On the other hand, the development of a rational method of design would provide some indication of the safety factor being employed. This in turn should lead to worthwhile pavement economy, because

of the greater confidence in the use of local materials, and the wider selection of aggregates suitable for bituminous mixtures, it should make possible.

## CONCLUSION

The present commonly employed empirical tests, Hubbard-Field, Marshall, and Hveem Stabilometer, appear to be fundamentally incapable of providing an adequate method of design for the stability of bituminous pavements and of expressing this stability on a unit strength basis. It has been the primary purpose of this paper, therefore, to attempt to outline a rational approach to this problem, based upon the triaxial test. The paper endeavours to examine the fundamental principles on which a rational method of design for the stability of bituminous paving mixtures might be based, rather than to provide finalized detailed design charts, since sufficient experimental data for some of the important variables involved are not yet available for the latter.

The triaxial test measures two fundamental characteristics of bituminous paving mixtures, cohesion c, and angle of internal friction Ø. It is the ultimate objective of a rational method of design for bituminous pavements based upon this test, to determine the lowest corresponding values for c and Ø that a bituminous mixture must possess, if it is to provide adequate stability for the particular conditions that pertain to any given project.

This paper has indicated that when all other factors are equal, the stability of a bituminous pavement seems to be materially dependent upon:

(1) lateral support L provided by the pavement adjacent to the loaded area
(2) pavement thickness
(3) shape of the curve of pressure distribution on the contact area
(4) influence of rate of loading or rate of strain on the value of cohesion c, and possibly on the value of the angle of internal friction Ø
(5) frictional resistance between pavement and tire
(6) frictional resistance between pavement and base
(7) stability in the direction of the longitudinal versus the transverse axis of the tire contact area
(8) whether the wheel loading is applied as a stationary load, as a load moving at a relatively uniform rate of speed, or as a load that subjects the pavement to severe braking or acceleration stresses

The three empirical tests, Hubbard-Field, Marshall, and Hveem Stabilometer, merely attempt to measure the stability of the bituminous mixture itself and are incapable of furnishing data that could be employed to explain why any one of the eight factors just enumerated should influence the stability developed by a paving mixture under traffic. On the other hand, if the development outlined in this paper is even qualitatively correct, the triaxial test appears to be capable of providing fundamental information that makes it a reasonably simpler matter to understand why each of these eight factors should have a marked influence on the stability that a paving mixture will develop in the field.

It should be particularly noted that each of these eight factors influences the stability of the bituminous mixture after it is in place in the pavement. Consequently, the stability value indicated for a bituminous mixture itself by one of the empirical stability tests, may afford little indication of the stability that the same mixture will develop in a pavement in service. Thus, a paving mixture that shows high stability when tested by one of the three empirical methods in common use, Hubbard-Field, Marshall, and Hveem Stabilometer, may actually develop instability in the field due to the influence of one or more of the eight factors listed, e.g. unusually low frictional resistance between pavement and base. On the other hand, a paving mixture that shows low and even inadequate stability when tested by one of these three empirical methods, may develop quite satisfactory stability in the field, because, on the particular project on which it happens to be laid, one or more of these eight factors may materially augment its stability, e.g. unusually high frictional resistance between pavement and base.

Furthermore, due to the influence on pavement stability of the lateral support L provided by the pavement adjacent to the loaded area, it was pointed out in connection with Figure 44 that any empirical stability test that is closely related to the unconfined compression test could not avoid indicating adequate stability for certain bituminous mixtures that would be unstable in the field, and rejecting other bituminous mixtures that would have adequate stability under field conditions.

Consequently, it would appear that little advance in the design of bituminous pavements and of our understanding of their performance in the field can be expected, as long as stability tests are employed that put the emphasis on the stability of the mixture itself as determined in the laboratory, instead of placing this emphasis where it belongs, namely, on the stability that the pavement must develop under the various conditions of stress, etc., to which it will be subjected in service.

The stability equations and illustrative stability diagrams included in the paper endeavour to indicate quantitatively the direction in which the cohesion c and angle of internal friction Ø for bituminous mixtures must be modified to provide either greater or less stability for the particular conditions of design associated with any given project. These stability equations and diagrams show that it should be quite possible to design bituminous pavements for either airports or highways that would be stable under the heaviest wheel loads and with tire pressures up to 300 to 400 p.s.i., or more. However, they also indicate that the design of bituminous mixtures for these high tire pressures can be expected to require considerably more care than has been given to this matter for the much lower tire pressures in use today.

It is realized that the approach to a rational method of design for bituminous paving mixtures outlined in this paper may have over-simplified the solution at some points, and that further refinements may be desirable. It may have disregarded certain factors that should be included and may have barely mentioned others that should receive more emphasis. On the other hand, limitations of space prevent reference to every detail, and the paper has been confined to a discussion of what appear to be the more important principles requiring consideration. It is appreciated that there may be quite other approaches to the rational design of bituminous paving mixtures, that could provide either equally or more accurate solutions, and it is hoped that the search for these will be stimulated.

## SUMMARY

(1) Some of the fundamental inadequacies of the empirical tests commonly used for the design of bituminous mixtures at the present time are mentioned, and the need for the development of a rational method of design, which would evaluate the strengths of bituminous mixtures on a unit strength basis, is pointed out.

(2) The open triaxial test, which is widely used on this continent, is described, together with the method of plotting the triaxial data in the form of a Mohr diagram.

(3) From the geometrical and trigonometrical relationships of the Mohr diagram, a general equation for the stability of bituminous mixtures is developed.

(4) A method for evaluating the lateral support L provided by the pavement adjacent to the loaded area is outlined.

(5) A description is given of the use of the cell triaxial test for determining the influence of the viscous resistance of a

bituminous mixture on its stability, and for measuring the effect of the rate of strain on the value of the cohesion c, and possibly on the angle of internal friction $\phi$, obtained for a bituminous mixture.

(6) The increase in the stability of a bituminous pavement due to frictional resistance between pavement and tire and between pavement and base, is indicated.

(7) The influence of braking and acceleration stresses on the design of bituminous pavements has been considered.

(8) A study of the influence of pavement thickness on pavement stability has indicated that a thick pavement should be designed to have higher minimum stability than a thin pavement for stationary wheel loads, and for wheel loads moving at a uniform rate of speed. On the other hand, for pavements subject to severe braking stresses, it appears that the influence of pavement thickness on pavement stability depends upon whether the bond between pavement and base is weak or strong.

(9) It is shown that the shape of the curve of pressure distribution on the contact area may materially influence pavement stability.

(10) It is demonstrated that the unconfined compression test, and all similar tests, can be quite misleading, insofar as indicating the stability that bituminous mixtures can develop in the field is concerned.

(11) The paper contains a brief discussion of the stability criteria to be considered when designing bituminous pavements for stationary wheel loads, for wheel loads moving at a uniform rate of speed, and for wheel loads that subject the pavement to severe braking stresses.

(12) Stability equations incorporating the different variables that seem to influence the stability of bituminous pavements have been developed, and illustrative stability diagrams, based upon these equations, have been included.

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## Discussion

MR. FRANCIS L. MARK: There was some criticism of various tests in the paper just presented. An immersion-compression test for determining the influence of water on the strength of bituminous mixtures has been adopted by the A.S.T.M. The rate of deformation is 0.2 inch per minute for a four by four inch cylindrical specimen. I want to ask Dr. McLeod if that could be demonstrated to be out of line.

DR. McLEOD: That would seem to depend upon the purpose to be served by the data obtained from the test.

MR. MARK: It is not intended for design, but is a test for determining the loss of cohesion of a bituminous mixture when exposed to water. I wanted to ask whether 0.2 inch per minute would be considered a logical rate of deformation.

DR. McLEOD: If the test procedure is merely to determine the loss of cohesion due to water action, the rate of deformation employed may not be too important. However, if the resulting data are to be considered as a basis for expressing the stability of bituminous pavements in service, as pointed out in the paper itself, the rate of deformation employed for testing the paving mixture would seem to depend upon the nature of the traffic to be carried by the finished pavement. For example, a different rate of deformation would seem to be justified when testing bituminous mixtures that will always be carrying fast moving traffic, as on rural highways, than for pavements subjected to a high percentage of more or less stationary wheel loads.

MR. CAMPEN: Dr. McLeod, by your diagrams and formulas, you predict certain resistance to displacement or certain stabilities. These are theoretical. Has this actually been proven? Has any field work been done at all to show that these calculated results are obtained? You say that by this formula you can design a bituminous pavement 3 inches thick, that will carry a wheel load of 400 p.s.i. Has that been demonstrated?

DR. McLEOD: We are quite dissatisfied with all the present empirical tests for evaluating the stability of bituminous mixtures. This paper, therefore, presents the results of a study we have made of the various factors that seem to influence the stability of bituminous pavements in service. On the basis of this study, we believe that pavement stability can be evaluated by means of the triaxial test. This study has been essentially theoretical, as Mr. Campen points out, but we have considered it to be quite necessary as a guide to the direction our practical tests should take in an attempt to measure pavement stability. Without such a theoretical study to point the direction which further investigational work should take, we will continue to flounder around with the increasing number of empirical tests that are being advocated and will make little or no progress toward a rational method of design. So far we have not utilized the triaxial test for actual pavement design, but we expect to do so very soon.

MR. CAMPEN: Well, has anyone demonstrated that your method can be employed?

DR. McLEOD: The Asphalt Institute's Manual on Hot-Mix Asphaltic Concrete Paving outlines a triaxial method for the determination of pavement stability for specification A-2-b. It also contains a diagram which indicates the corresponding values of c and Ø required for adequate stability. In his paper for last year's A.A.P.T. meeting, Mr. Vaughn Smith outlined the theoretical and experimental background for The Asphalt Institute's stability diagram based upon the triaxial test. Mr. Smith gives data to show that his triaxial method has been correlated with field experience.

The Asphalt Institute's stability diagram was established on a different basis than that outlined in our paper, and may eventually be modified in a number of respects. Nevertheless, I believe that the triaxial method described by The Asphalt Institute, and the stability diagram accompanying it, is quite conservative and can be safely used for asphalt pavement design for heavy duty traffic for highways and city streets, provided the pavement is firmly bonded to the base course.

MR. MERRILL: Dr. McLeod, I think I can agree with you in one respect, when you state that the coefficient of friction between the pavement and base is of considerable importance. We have found that with a bituminous concrete pavement on a crushed rock base, having what you might say is a mosaic top, that we have had better success than when we have laid the pavement on a smooth concrete base.

MR. G. L. OLIENSIS: I have been listening with keen interest to Dr. McLeod's very able presentation of a difficult and highly involved subject to which it is clear he has devoted much thought and time and study. One question, however, that has always been in my mind is whether the importance of temperature has been sufficiently taken into consideration in formulae of the type that he has developed, inasmuch as fluctuations in temperature may be capable of introducing rather disturbing factors in the stability and wear resistance of bituminous pavements.

DR. McLEOD: Insofar as the triaxial test or any similar test is concerned, it is a very simple matter to control the temperature at which the test is made. For his investigations in Holland, Nijboer employs a temperature of 120° F. for the triaxial testing of bituminous mixtures, because 120° F. is the highest pavement temperature normally observed in Western Europe. On this continent, the testing of bituminous mixtures at 140° F. is common practice.

MR. OLIENSIS: But the question I have in mind is this: Let us assume that you have developed a bituminous paving mixture that shows excellent resistance to abrasion and impact and heavy loading at the test temperature of 120° F. It is quite likely that to obtain such results, particularly a high resistance to deformation under prolonged heavy loading, you will have to have recourse to asphalts of low penetration. We know that in India, for example, with its extraordinarily high temperatures, they have used unfluxed Trinidad refined asphalt, which at 77° F. has a penetration of only 3 or 4 and is definitely hard and brittle, as the binder for their mineral aggregate in asphalt pavements, and have had marked success with it. Yet we know that such a pavement laid with the same hard Trinidad asphalt in colder climates like that in our Northern tier of states or in Canada, would rapidly crack and eventually go to pieces.

Now we know that many airfields will be subjected both to extremely high summer temperatures and extremely low winter temperatures, and yet throughout all these wide ranges of temperature they will have to be able to stand the same heavy loading and the same intense pounding and shearing and abrasion stresses. What can be done when a pavement, whose composition or structure has been modified with the express purpose of giving it maximum resistance to impact and prolonged heavy loading at average temperatures of 120° F., is found by virtue of those very modifications to be too susceptible to cracking and ravelling at low temperatures? What modifications in your formulae can be introduced to take care equally satisfactorily of all those widely divergent weather conditions which every airfield must at one time or another undergo?

CHAIRMAN BENSON: Could we infer from your equations that there may be other factors which may enter, which are not included in your equations but which may modify them in accordance with some factors mentioned by Mr. Oliensis.

DR. McLEOD: I don't know. Possibly we should hand this question back to Mr. Oliensis and ask him what he would recommend.

More seriously, however, it should be possible to handle this problem in the manner recommended by Mr. Prevost Hubbard and others a number of years ago, namely, to employ the softest asphalt binder and the largest percentage of binder that the paving mixture can tolerate without decreasing its stability below the critical minimum required for satisfactory service performance. This would provide the flexibility needed to avoid cracking at low temperatures, and the minimum stability required at the highest summer temperatures. We believe that the triaxial test will be found to be a much more useful method for designing bituminous mixtures on this basis, than those involving our current empirical tests.

CHAIRMAN BENSON: The factors mentioned by Mr. Oliensis may have a great deal of bearing on the actual service behavior of pavement. The definition of failure itself is sometimes very obscure and indefinite. It is probable that the evaluation of many factors, including those mentioned by Mr. Oliensis, must eventually be considered if we are to evaluate extremes of conditions.

MR. ROLAND VOKAC: There is one loophole I would like to have Dr. McLeod plug up. If you take Ottawa sand and mix it with asphalt, you would have what we think of as a rather unstable mix. Nevertheless, if we lay that mixture thin enough it won't shove under the traffic. This is evidenced in our natural rock asphalt mixes. Dr. McLeod called attention to the frictional resistance between the pavement and the base as contributing additional lateral support that he found necessary to account for on the basis of the classical theory used. It seems to me that the classical theories don't quite fit the picture. Here is a mixture that is unstable, say if we lay it out four or five inches thick, but it is stable if we put it down half or three-quarters inch thick. I don't think that is entirely accounted for by the frictional resistances involved. I believe that there are some dimensional effects that haven't been adequately taken care of in the application of the theories that are involved and the derivation of their equations. I would appreciate it very much if Dr. McLeod could plug up that loophole.

DR. McLEOD: It seems to me that some of the slides covered the points that Mr. Vokac appears to have in mind. They have shown that the frictional resistances between pavement and tire and between pavement and base explain the observations of pavement performance he has described.

MR. VOKAC: I still don't believe that we have gotten quite to the "nub" of the question, which I will try to state quite simply.

The real question that I have in mind is simply the fact, that given a mixture for which you measure certain definite characteristics, namely cohesion and angle of internal friction and, without changing those characteristics one iota, I can lay a pavement one inch thick, that will not rut or shove, and if I lay up the pavement four inches thick it will rut and shove, yet I have not changed any of the constants that were assigned to the mixture by the test.

That is why I feel that there must be a very significant dimensional effect involved here and we cannot ignore the dimensions of the test specimen any longer in the use of Triaxial Test data. I have that conviction dating back about ten or fifteen years to my own work with the "unconfined" compression testing, whereby I found that there was a tremendous difference between testing a specimen that was taller than it was broad and testing one that was, say about four times as broad as it was high. There are some dimensional relationships involved, whereby your fundamental theories do not embrace the entire dimensional range that is being acted on beneath the tires; and, incidentally, I believe we, all of us, recognize today that our pavements are being loaded, not so much as long, slender cylinders, but rather as flat discs beneath each tire.

CHAIRMAN BENSON: Dr. McLeod, do you have a short rebuttal to Mr. Vokac's remarks?

DR. McLEOD: I am inclined to feel that when Mr. Vokac has had the opportunity to study our paper in detail, he will find that the dimensional relationships to which he refers have been covered much more completely than it has been possible for me to describe in the short time that has been available for its presentation here.

MR. ROLAND VOKAC (by letter): *In further elaboration on* my comments regarding dimensional effects I should like to refer to a paper of mine published in the 1937 proceedings of the American Society for Testing materials, Vol. 37, Part II, pp. 509-516 including the discussion of W. S. Housel on pp. 517-518. The title of this paper is "Compression Testing of Asphalt Paving Mixtures — II." This paper reports on a study of the flow properties of asphalt mixtures interpreted from "unconfined compression test" data.

The ordinary concept of plastic flow comprising "mobility" and "yield value" obtained from measurement of the rate of shear and shearing stress is described. Then it is pointed out that, — "the rate of shear in any present day method of viscosity (or mobility) measurement is always a proportional function of the rate of volume displacement of the sample in the test." Also, "since this relationship appears to be fundamental to measurement of flow — ," the following analogy was considered feasible:

1. The rate of volume of material displaced in a compression test is easily measured by the rate of compression of the sample. This unit rate of volume displacement may be considered "analogous and most probably a direct function

of the rates of shear existing in a specimen during test." Thus,

$$R = \frac{vA}{h} \tag{1}$$

R = unit rate of volume displaced in cu. in. per inch per minute
v = velocity of compression of the sample in inches per minute
h = original height of specimen in inches

2. Carrying this line of analogy further it is recognized that compressive stress may always "be resolved into two components originating from shear resistance in the material." Therefore, the compressive stress is some direct function of the shear stress in an asphalt mixture.

Based on these two considerations of analogy, the equation:

$$p = aR + K \tag{2}$$

was developed, where

p = compressive stress in lbs. per sq. inch.
a = a constant analogous to and representing the "mobility" of a mixture.
K = a factor analogous to and representing the "yield value" or "yield strength."

Testing these assumptions by an elaborate series of samples of a given mixture composition shows excellent agreement with the data obtained when the rate of volume displacement is varied at will and the diameter/height ratio $\left(\frac{D}{h}\right)$ is held constant. Samples tested were 1.13, 1.598 and 2.26 inches in diameter molded as close to a diameter/height ratio = 2.00 as possible. Actually $\frac{D}{h} = 2.035 \pm 0.042$. This amounts to a deviation of only 2.06% and really shows good control. The samples were tested at rates varying from 0.0076 to 0.150 inches per minute according to size to obtain values of R between 0.02 and 0.28 cubic inches per inch per minute for each of the three diameters of samples used. The data from testing this series show:

$$p = 1090R + 662 \tag{3}$$

with an average deviation of only 5 2/3 per cent.

Another set of briquetts of the same mixture were made and tested varying the $\frac{D}{h}$ ratio from 0.50 to 2.00 and R between 0.045 and 0.190. Values of K were calculated according to Equation 3, by subtracting 1090R from the data value of p. These K-values were found to vary proportionally to the $\frac{D}{h}$ ratio according to the equation

$$K = b\frac{D}{h} + c \qquad (5)$$

and evaluating the constants according to the data

$$K = 292\,\frac{D}{h} + 106 \qquad (6)$$

with an average deviation of only 4.85 per cent.

The general equation for plastic flow is therefore

$$p = aR + b\frac{D}{h} + c \qquad (7)$$

and specifically for this mixture

$$p = 1090R + 292\,\frac{D}{h} + 106 \qquad (8)$$

The facts discovered and especially pertinent to the present discussion are:

1. For any selected ratio of diameter to height, the compressive strength of an unconfined compression test sample is directly proportional to the rate of volume displacement in the test.
2. The "mobility constant" (a) in equations 2 and 7 is not affected by the dismensions of the test specimen. It is apparently a true function of the testing rate of volume displacement and so analogous to the mobility or flow characteristic of the material tested.
3. The value of K, the "yield value" so called, is the property affected by dimensional variation expressed by the $\frac{D}{h}$ ratio when the "mobility" has been properly evaluated.

Quoting again from the aforementioned paper, — "This is equivalent to saying that although varying the dimensions of the specimen will change its 'yield value,' such variation will have no effect whatsoever on the "mobility" of the mix." More specifically and important to our discussion, — "Evidently the dimensional relationship $\frac{D}{h}$ largely determines the ability of a mixture to withstand loading without deformation."

In conclusion, and returning to the specific question of our discussion on the floor of the meeting, there should be some similar reasoning that may be applied to the triaxial data to show:

1. The fundamental characteristic of flow, which perhaps is properly measured in the factor of cohesion (c) in your formulas and as suggested by you. I wonder if it will be found to vary greatly at various rates of volume displacement. In other words, is it similar to "p," the compressive strength, or is it more like "a," the mobility constant, in the equations I have listed.
2. The large effect of dimension on a "yield value" such as our factor "K" which indicates that the critical point where flow commences will be much higher for flat, disc-like shapes than with taller, cylindrical specimens. These are the only data I have ever seen presented that indicate a means to measure the phenomenon of common experience, i.e., that a thin layer of asphalt mixture is more "stable" than a thick layer under traffic.

In all sincerity I wish to assure you of my great respect for the volume of work you have done and your ability to analyze and present your facts. If in this discussion I have been able to generate a clue in your mind whereby you will see greater possibilities in your data, I will be most happy for your successful accomplishment. You will help not only to clarify my own difficulty but also eliminate one of the largest obstacles in the way of general acceptance of the tri-axial method of evaluating asphalt mixtures in the design of pavements.

DR. McLEOD (by letter): After making some careful study of Mr. Vokac's written discussion, we believe that the equations he has presented, on the basis of his work with the unconfined compression test, and the conclusions he has drawn, provide further support for the development outlined in our paper, rather than otherwise.

For example, examination of his equation (2),

$$p = aR + K \qquad \text{(2) Vokac}$$

indicates that his constant K must represent the unconfined compressive strength of the mixture with which he was working, when the rate of strain at which the mixture was tested was zero. That is when $R = 0$, it is quite clear that $p = K$.

The corresponding relationship for the unconfined compression test, as illustrated by the Mohr diagram is shown in the accompanying Figure 45. This demonstrates that Mr. Vokac's K is the value of the unconfined compressive strength when the rate of strain is zero, and that the quantity "aR" from his equation (2) is the increment in the value of the unconfined compressive strength that results from the influence of any given rate of strain at which a bituminous mixture is tested.

Mr. Vokac's equation (2) indicates a straight line relationship between unconfined compressive strength "p" versus rate of strain "R". Figure 45 demonstrates that a similar straight line relationship between the value of the unconfined compressive strength versus rate of strain could not be expected unless there was a linear relationship between cohesion "c" from the Mohr diagram versus rate of strain. Figure 24 of the paper by Goetz and Chen on "Vacuum Triaxial Technique Applied to Bituminous Aggregate Mixtures," in this volume, indicates that a straight line relationship

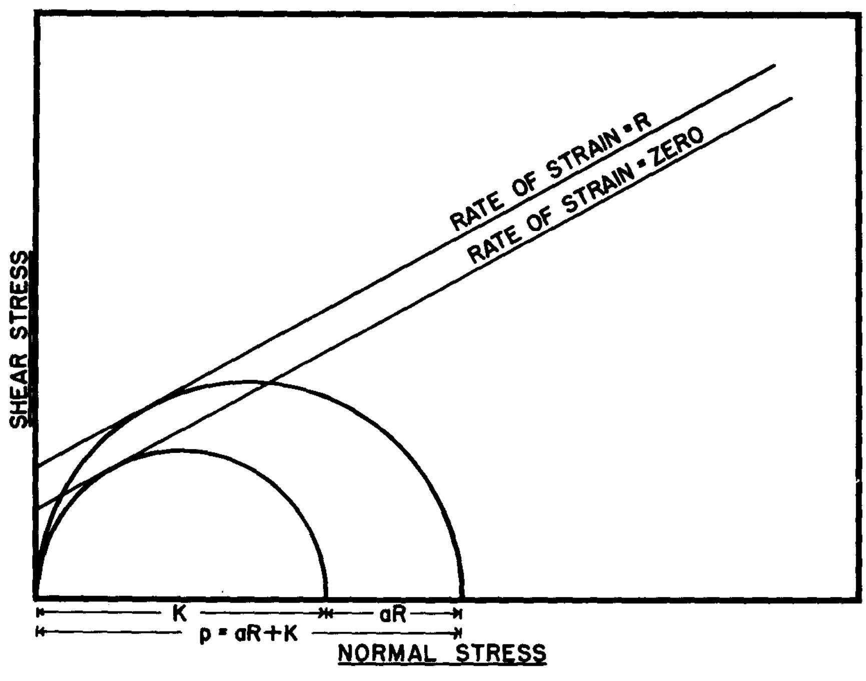


Figure 45. Influence of Rate of Strain on Unconfined Compressive Strength of Bituminous Mixtures

for cohesion "c" versus rate of strain cannot be expected over a wide range of strain rate, but might hold over a limited range.

Mr. Vokac's discussion next points out that from his further unconfined compressive strength data obtained from the testing of specimens of the same bituminous mixture, but with different ratios of diameter to height, that the value of K was found to depend directly upon the diameter/height ratio $\left(\frac{D}{H}\right)$ of the test specimen. This conclusion is expressed in Mr. Vokac's equation (5),

$$K = b\frac{D}{H} + c \qquad \text{(5) Vokac}$$

From Mr. Vokac's equation (5), it is clear that for a specimen with $\frac{D}{H}$ ratio equal to zero, that is, for a specimen that is very tall relative to its diameter, equation (5) reduces to $K = c$. This would be the case for specimens that are sufficiently tall relative to their diameter, that the constraint at the ends during testing would have no influence on the compressive strength value obtained. Therefore the quantity $b\frac{D}{H}$ on the right hand side of Mr. Vokac's equation (5) represents the increase in compressive strength that results from testing specimens that are so short relative to their diameter, that this dimension ratio has a measurable effect, and in the case of quite short specimens, a very marked influence on the compressive strength value. It is further apparent that Mr. Vokac's equation (5) represents a straight line relationship between K and the ratio $\frac{D}{H}$.

It is of particular interest to us that, as indicated in his equation (7), Mr. Vokac found that the compressive strengths of the mixture he investigated varied only with the rate of strain he employed, and with the $\frac{D}{H}$ ratio of his test specimens. Apparently, even for his shortest specimens, he found no columnar effect, that is, no tendency for direct transfer of load through the specimen from top to bottom bearing plates.

We are particularly interested in Mr. Vokac's equations and comments in this respect, because in the development presented in our paper we have also pointed out the great influence of rate of strain, and of $\frac{D}{H}$ ratio, on the strength or stability developed by a bituminous mixture.

We have not directly referred to the $\frac{D}{H}$ ratio in our paper, but we have clearly pointed out that the increase in pavement stability due to frictional resistance between pavement and tire and between pavement and base is greatly influenced by the width or length of the contact area and the thickness of the pavement, e.g. Figs. 22, 23, 24, 25, 26, 43, etc. Width or length of the contact area, and pavement thickness, correspond to Mr. Vokac's diameter D and height H, respectively.

We have not had time so far to determine whether or not the effect of frictional resistance between pavement and tire and between pavement and base would result in the straight line relationship between K and $\frac{D}{H}$ indicated by Mr. Vokac's equation (5). However, it will be observed that the stability curves of Figs. 22, 23, 24, and 25, which are based on various values for frictional resistance between pavement and tire and between pavement and base, indicate a straight line relationship between stability versus $\frac{D}{H}$ ratio for the conditions pertaining to these figures.

We feel, therefore, that Mr. Vokac's comments serve to support the development outlined in our paper, since his comments and our paper both emphasize the important influence of rate of strain and of the ratio of size of contact area to pavement thickness, on pavement stability. However, Mr. Vokac's comments do not attempt to explain <u>why</u> the ratio of the size of contact area versus pavement thickness $\left(\frac{D}{H}\right)$ should influence pavement stability. In our paper on the other hand, we have indicated that this seems to be due to the effect of the frictional resistance between pavement and tire and between pavement and base. We have also shown that consideration of these two frictional resistances contributes materially to a rational explanation for the various types of pavement instability and distortion observed in the field.

Near the end of his discussion, Mr. Vokac states: "The large effect of dimension on a "yield value" such as our factor K which indicates that the critical point where flow commences will be much higher for flat, disc-like shapes than with taller, cylindrical specimens (is shown). These are the only data I have ever seen presented that indicates a means to measure the phenomenon of common experience, i.e., that a thin payer of asphalt mixture is more "stable" than a thick layer under traffic."

It was probably not clear from the presentation of our paper at the meeting itself, that a consideration of the influence of frictional resistance between pavement and tire and between pavement and base on pavement stability will do exactly that to which Mr. Vokac has referred in the final sentence of the comment just quoted. Our Figs. 31, 32, 37(a), (b), (c), and (d), and 43 demonstrate that by means of these two frictional resistances it is a simple matter to explain why "a thin layer of asphalt mixture is more "stable" than a thick layer under traffic."

As our paper also indicates, by considering the influence of frictional resistance between pavement and tire and between pavement and base on pavement stability, it is possible to go much further than Mr. Vokac has indicated in explaining the effect of pavement thickness on pavement stability.

We have shown that for pavements subject to stationary loads, or to loads moving at a uniform rate of speed, consideration of the influence of frictional resistance between pavement and tire and between pavement and base on pavement stability indicates that thick pavements must be designed to have a greater minimum stability than thin pavements to carry a given wheel load, e.g. Figs. 31, 32, 37(a), (b), (c), and (d). This is in keeping with Mr. Vokac's remarks.

On the other hand, for pavements subject to braking stresses, consideration of the influence of these same two frictional resistances on pavement stability indicates that thin pavements must be designed to have a greater minimum stability than thick pavements, if there is a poor bond between pavement and base, e.g. Fig. 43. It also indicates that pavement thickness has no influence on pavement stability under severe braking stresses, if the frictional resistance between pavement and tire is equal to that between pavement and base ($f - g = 0$). It indicates further that, if there is a very strong bond between pavement and base, $f - g < 0$, a thick pavement must be designed to have a higher minimum stability than a thin pavement. In the two latter cases, however, the tendency of the load to squeeze the pavement out from under the contact area may provide a more critical criterion for design than the influence of the braking stresses themselves. It seems unlikely that these latter deductions concerning pavement design for the condition of severe braking stresses, could be derived from the conclusions from his work with the unconfined compression test summarized in Mr. Vokac's discussion. Consequently, the development based on the triaxial test outlined in our paper, not only offers a reasonable explanation for each of the bituminous pavement performance characteristics referred to in Mr. Vokac's discussion, but for a considerable number of other factors having an important influence on the design and service behaviour of bituminous pavement as well. These factors have been listed in summary form under the sub-heading, "Conclusion," in our paper.

For the reasons that have just been outlined, therefore, we believe that Mr. Vokac's remarks tend to generally support the development outlined in our paper and we would like to thank Mr. Vokac for the considerable trouble to which he has gone in preparing his discussion. He has carried on a great deal of excellent investigational work over the years, and has acquired a wide background of experience in both the laboratory and the field. Consequently, his comments are always instructive and greatly appreciated.

MR. WALTER C. RICKETTS[1] (by letter): Mr. McLeod has made an excellent presentation of an approach to a theoretical method for the design of bituminous paving mixtures based on the triaxial test. The description of the triaxial apparatus and the reference to investigators of the subject indicates that Mr. McLeod refers to the triaxial machine and method described in two papers[2,3] by V. R. Smith presented in 1949 before the Association of Asphalt Paving Technologists and American Society for Testing Materials or similar method based on the triaxial test.

It is considered that the paper merits discussion for the following reasons:

1. To point out that many of the charts and equations are based on a number of assumed design factors with values which are yet to be determined by laboratory and and field work.

2. The criticism that all existing testing machines and methods for designing bituminous pavements, particularly Hubbard-Field, Marshall and Hveem machines and design methods are inadequate, misleading, and confusing even to experienced design engineers.

The purpose of this discussion is to show that:

1. Considerable laboratory work and correlation with pavement performance is required before a bituminous pavement design based on the triaxial test and the numerous charts and equations shown could be considered to be even a satisfactory approach as indicated by Mr. McLeod.

2. Mr. McLeod's criticism regarding the inadequacy of existing machines and methods, particularly Hubbard-Field, Marshall,

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[1] Office, Chief of Engineers, Washington, D.C.

[2] V. R. Smith, "Triaxial Stability Method for Flexible Pavement Design," Proceedings A.A.P.T., Volume 18 (1949).

[3] Preprint, V. R. Smith, "Application of the Triaxial Test to Bituminous Mixtures, "A.S.T.M., San Francisco, California, October (1949).

and Hveem, to properly design bituminous pavements is unwarranted and is not substantiated by experience.

3. Information for design of bituminous pavements taken from the figures and equations may be misleading and technically incorrect as they are based on design factors with undetermined values.

## PRESENT STATUS OF TRIAXIAL TESTING OF BITUMINOUS PAVING MIXTURES

A committee known as the "Triaxial Institute" composed of representatives of the State Highway Departments of Washington, Oregon, and California, Universities of Washington and California, California Research Corporation, Shell Development Company, headed by Mr. V. A. Endersby, Shell Development Company, Emeryville, California, was formed to conduct research work and develop a suitable method for designing bituminous pavement based on the triaxial test.

There are listed below quotations from preprints of two papers by Messrs. Endersby and Smith, both members of the Triaxial Institute, presented at recent A.S.T.M. meeting in San Francisco, California.

### Paper[4] by V. A. Endersby

1. "The reasons for the formation of the Institute are discussed and research of the fundamental problem of forming test specimens to properly represent materials laid in pavements is described."

2. "Outstanding Questions. There are various outstanding questions regarding the triaxial test that need further investigation:

a. What is the most acceptable interpretation for use by the routine testing engineer?

b. What is the effect of low height/diameter ratio prevalent in the field under road loading conditions?

c. What is the real meaning of intercept which denotes static or initial resistance in the Mohr diagram? This is usually called "cohesion" but shows some properties not compatible with the definition, such as in some mixes decreases in its value with decreasing friction.

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[4] Preprint, V. A. Endersby, "The History and Theory of Triaxial Testing and the Preparation of Realistic Test Specimens, "A report of the Triaxial Institute A.S.T.M., San Francisco, California, October (1949).

d. What is the proper method of compacting samples? (This point is so important that we devote a special section to it.)"

3. "Basically the Institute was formed with two objects: First to determine to the satisfaction of a group of representative interests the fundamentals of a test which would be fully scientific in principle and whose results could be quickly interpreted in terms suitable for routine laboratory; second, to bring about as complete correlation as possible with other tests, it being well recognized that long established tests will not be readily abandoned even though scientifically superior ones can be established.

4. "The Problem of Realistics Test Specimens. The first meeting brought out a definite consensus that a very primary problem in the general field has not been solved. That is, most current methods of fabricating test specimens did not reproduce the properties of field mixes.

5. "We believe that any method, however inexpensive, which produces a material for testing which is radically different from that actually placed in the field is unacceptable. The Committee departs from the widely held view that the testing method is the thing and that methods of fabrication are more or less incidental and unimportant. While our cooperative physical research has not proceeded far as yet, we can show very striking evidence in results from the laboratories of some of the members of the Committee."

## Paper[5] by V. R. Smith

1. "All stability tests now in use including triaxial will yield differences in measured stability values when different specimen compaction procedures are employed. This matter deserves maximum emphasis."

2. "Kneading type compaction is known to yield specimens approximating closely the particle orientation and stability properties obtained in actual field construction. This matter certainly deserves further study and such studies now are being undertaken by various interested groups."

3. "Figure 4 shows the results of tests on hot plant mixes, road mixes (both machine mixed and blade mixed) and sheet asphalt mixes employed by various highway departments throughout the United States. All of these mixes were made strictly according to the specifications and construction practices of these highway organizations and are considered satisfactory for the traffic conditions encountered. The traffic conditions represented vary from moderate with the road mixes to heavy with the sheet asphalts and

[5] Preprint, V. R. Smith, op. cit.

hot plant mixes. Except in a few instances first hand observation of the performance properties of these surfaces has not been possible."

4. "Experience with the evaluation charts presented in Figures 4 and 5 is limited. Possibly these charts will have to be revised after more extensive data on service behavior are obtained."
NOTE: Figures 4 and 5 indicate mixes considered to be satisfactory and unsatisfactory for heavy traffic and light traffic based on values of c, unit cohesion, and Ø, angle of internal friction.

5. "NOTE: (Forming of test specimens having properties closely simulating those obtained in field construction is essential in procuring precise triaxial test results. At the present time several road material laboratories have under development improved briquet compaction procedures which promise to supersede current methods.)"

It is believed that the following comments are in accord with the quotations mentioned above:

1. The Triaxial Institute recognizes the need for research on a number of problems in connection with triaxial testing of bituminous mixtures and is planning the necessary experimental work to solve the problems.

2. The problem of forming test specimens to properly represent materials laid in pavements is a high priority item as most of the current methods do not produce test specimens comparable with the properties of field mixtures.

3. It will be necessary to formulate an interpretation of the results obtained by triaxial testing suitable for use by the routine testing engineer.

4. The work performed to date by Institute members indicates that triaxial test values on specimens prepared by different compactive methods are at variance.

5. The correlation of triaxial values for unsatisfactory and satisfactory bituminous mixes as recommended by Mr. Smith with service behavior of actual pavements is limited and Figures 4 and 5 are subject to change when more extensive data on service behavior are obtained.

It is to be emphasized that this discussion is not intended to be critical of the Triaxial Institute or any experimental work on triaxial testing in connection with bituminous pavements accomplished by its members or other investigators.

The objectives and the type of research planned by the Triaxial Institute as described in Mr. Endersby's paper are considered to be above criticism and technically sound for developing a design method for bituminous pavements based on the triaxial test.

## ANALYSIS OF MR. McLEOD'S EQUATIONS AND FIGURES

A careful study of a number of the equations and figures indicates that the charts and figures are based on exact values for c and Ø of paving mixtures for given loads with consideration being given to a number of other factors with undetermined values.

No consideration is given to a factor of safety to provide for vagaries which occur many times during preparation and laying of paving mixtures.

The method for determining the most suitable asphalt content for a given mixture to be used for a given design condition by use of the triaxial test is not described either in the text or the equations and charts.

The rational approach to the design for bituminous paving mixtures proposed by Mr. McLeod is based on triaxial testing which the Triaxial Institute recognizes as a method of test in need of considerable development and correlation with pavement performance before it would be of use to design engineers charged with the responsibility of providing adequate pavements.

A considerable number of design factors with either undetermined or assumed values have been used in the formulation of the equations and charts. Some of these are:

- L. Lateral support in p.s.i. developed by portion of pavement adjacent to the loaded area.
- K. Factor to take in account other sources of lateral support.
- J. Another factor to take in account other sources of lateral support.
- f. Coefficient of friction between pavement and tire.
- g. Coefficient of friction between pavement and base.
- P & Q. Ratios used in Equations 9, 10, 11, 12.

No attempt is made to list all the equations and figures in which various symbols with assumed or undetermined values have been introduced.

Figure 11 which is based on equation 3 with a value of L of 30 p.s.i. indicates that all paving mixtures to the right of curve labelled V = 100 p.s.i., L = 30 p.s.i. would have required stability and those to the left would be unsatisfactory. Here again is an assumption as engineering data is not offered to substantiate whether all mixtures considered satisfactory possess an L value of 30 p.s.i. Further no evidence of correlation with pavement performance is given which would indicate that all paving mixtures to the right of the curve are satisfactory for a V value of 100 p.s.i.

## HISTORY OF HUBBARD-FIELD, HVEEM AND MARSHALL TESTING METHODS

### Hubbard-Field and Hveem Testing Machines

The use of the testing machines developed by Messrs. Hubbard, Field and Hveem in designing many miles of satisfactory pavements for highways and city streets over a period of twenty years is well known to the members of the A.A.P.T. and any discussion as to their merit is considered unnecessary.

### Marshall Machine

The investigational work which lead to development of Marshall machine was initiated by the Corps of Engineers, Department of the Army, the later part of 1943. The work was directed toward the development of criteria to be used in constructing asphalt pavements suitable for various types of military airplanes.

The criteria established for adequate pavements were based on results of traffic tests on actual pavements using properly weighted airplane wheel assemblies.

The laboratory and field work on which the criteria are based is contained in Corps of Engineers' report[6] published in 1948.

The established criteria for asphaltic concrete pavements adequate for traffic of heavy airplanes consists of numerical values for the following properties:

1. Stability
2. Flow
3. Unit Weight
4. Per Cent Voids
5. Per Cent Voids Filled with Asphalt

The first two properties are measured by the Marshall machine machine, the remainder is calculated from values determined by standard laboratory tests. The criteria also include requirements for gradation and quality of aggregate and limits for percentage of mineral filler to be used in an aspahltic concrete pavement.

It was also determined from the investigation that a pavement conforming to the established criteria and having a Marshall stability of at least 500 pounds was satisfactory for the range of heavy airplane wheel loads used in the traffic tests.

[6] Corps of Engineers' report "Investigation of the Design and Control of Asphalt Paving Mixtures" in 3 Vol., dated May 1948.

The investigational work indicates that the stability values of asphaltic concrete pavements designed in accordance with the established criteria and present Corps of Engineers specifications are generally considerably higher than the minimum 500-pound Marshall stability required.

Corps of Engineers report[7] published in 1948 contains the supporting laboratory and field work on which the pavement criteria were established. The Marshall machine and established pavement criteria have been used successfully by engineering agencies other than the Corps of Engineers not only for design of bituminous pavements but for control of preparation and laying of mixtures in the field.

The Corps of Engineers is continuing investigational work on bituminous paving mixtures to provide pavements adequate for the traffic of all types of newly developed military airplanes.

## INFLUENCE OF AMOUNT AND TYPE OF AGGREGATE ON MARSHALL STABILITY VALUES

Mr. McLeod criticism that the stability values obtained by the Marshall machine are largely influenced by the cohesion of the mixture is not born out by the laboratory data plotted on Figure 46.

A number of specimens were prepared in which the percentages of 3/4" maximum sized aggregate retained on the No. 10 seive was varied in five per increments from 20 percent to 70 percent. The asphalt content used in each mix was the amount which produced maximum stability and various types of aggregate were used namely slag, crushed limestone, crushed gravel and uncrushed gravel.

It is apparent from the stability curves shown on Figure 46 that the Marshall stability values definitely measures internal friction. It is well known to experienced asphalt pavement designers that the coarse aggregate content of a mixture must exceed about 35 percent before the effect of interlocking of the coarse aggregate takes place. It is to be noted that:

1. The stability values begin to increase when the mixtures contain about 35 percent coarse aggregate for all types of aggregate.

2. The stability values of the specimens containing uncrushed gravel did not increase materially due to the lack of angularity of the aggregate.

3. The stability values of the specimens containing crushed

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[7] Ibid.

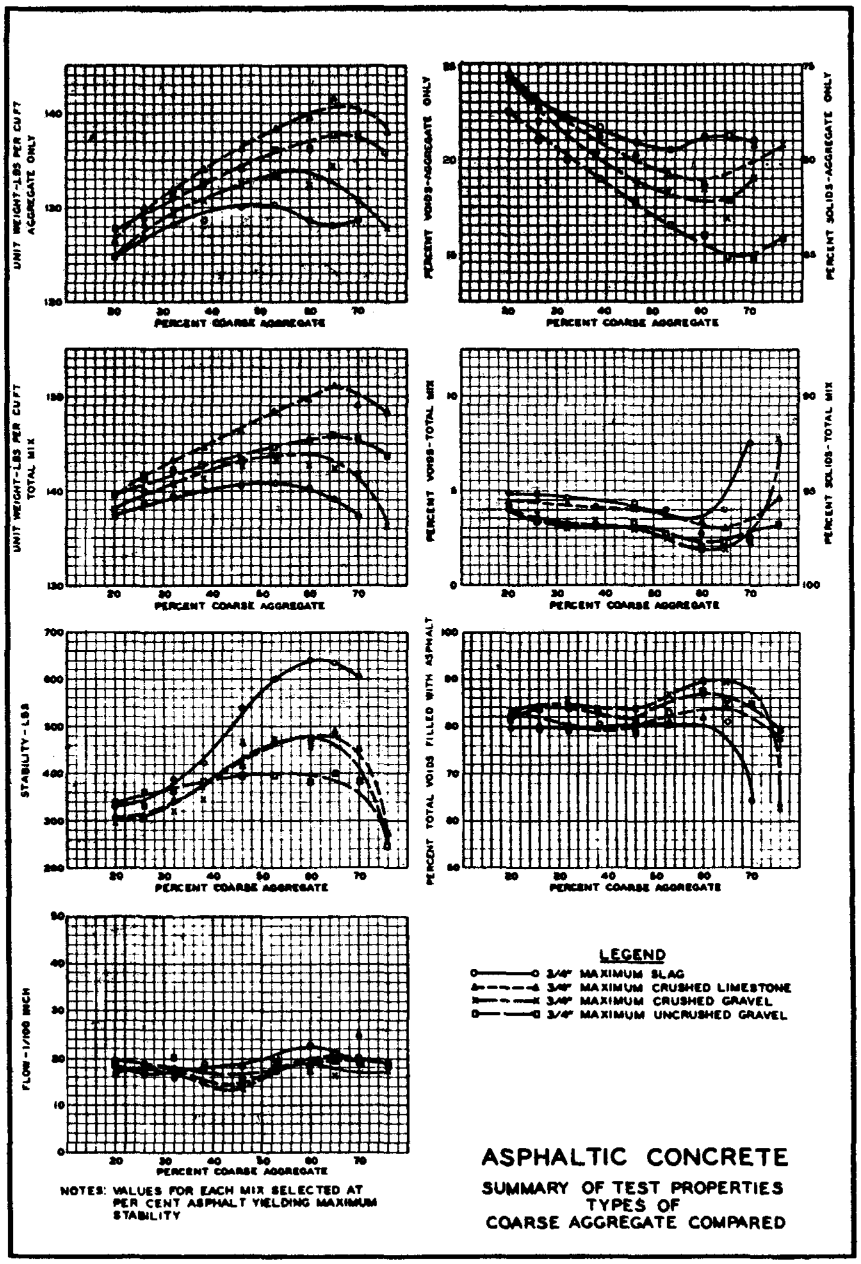
UNIT WEIGHT-LBS PER CU FT AGGREGATE ONLY
PERCENT VOIDS-AGGREGATE ONLY
PERCENT SOLIDS-AGGREGATE ONLY
UNIT WEIGHT-LBS PER CU FT TOTAL MIX
PERCENT VOIDS-TOTAL MIX
PERCENT SOLIDS-TOTAL MIX
STABILITY-LBS
PERCENT TOTAL VOIDS FILLED WITH ASPHALT
FLOW-1/100 INCH
PERCENT COARSE AGGREGATE
LEGEND
3/4" MAXIMUM SLAG
3/4" MAXIMUM CRUSHED LIMESTONE
3/4" MAXIMUM CRUSHED GRAVEL
3/4" MAXIMUM UNCRUSHED GRAVEL
NOTES: VALUES FOR EACH MIX SELECTED AT PER CENT ASPHALT YIELDING MAXIMUM STABILITY
ASPHALTIC CONCRETE
SUMMARY OF TEST PROPERTIES
TYPES OF
COARSE AGGREGATE COMPARED

Figure 46

gravel and crushed limestone increases in proportions to the percentage of coarse aggregate up to about 65 percent.

4. The stability values of the specimens containing slag are the highest due to the very irregular and angular characteristics of the aggregate.

5. The Marshall stability machine measures the interlock of the coarse aggregate fraction and stability values reflect the shape and quantity of the aggregate used in the specimen.

## ANALYSIS OF FIGURE 44 DEMONSTRATING THE INADEQUACY OF AN UNCONFINED COMPRESSION TEST FOR MEASURING THE STABILITY OF BITUMINOUS PAVING MIXTURES

Mr. McLeod makes the following statements in connection with the Marshall Test: "Figure 44 clearly demonstrates that the unconfined compression test and probably all similar tests such as the Marshall test, that are closely related to it are fundamentally incapable of providing trustworthy measurements of the stability that bituminous paving mixtures can develop under field conditions. Figure 44 (a) shows that the unconfined compression test is capable of indicating adequate stabilities for bituminous mixtures that would be unstable in the field. Figure 44 (b) demonstrates that the unconfined compression test is also capable of rejecting bituminous mixtures that would have adequate stability under field conditions." "Figure 44 (a) demonstrates that such changes could be made in the composition of bituminous mixtures which would have little or no effect on the unconfined compressive strength but that would at the same time greatly influence the stability of the pavement under the conditions of stress to which it is exposed in the field. On the other hand, Figure 44 (b) indicates that wide changes can be made in the composition of bituminous mixtures which would cause large variations in their unconfined compressive strength values but that would have little influence on pavement stability under field conditions."

A number of assumptions regarding Marshall values has been introduced in Figure 44. However only those assumptions considered to be particularly pertinent to the Marshall test are discussed.

Two of the assumptions introduced in Figure 44 are:

1. Three different mixtures represented by Mohr envelopes xw, yv and zu in Figure 44 (a) have the same unconfined compressive strength (Marshall stability value) for the corresponding value for c and $\phi$ indicated. Further, mixtures represented by yv and zu would be unstable for the vertical load indicated.

2. Three different mixtures represented by Mohr envelopes lr, mq and np in Figure 44 (b) have three different unconfined compressive strengths (Marshall stability values) for the corresponding values of c and Ø indicated. Further mixtures represented by lr and mq would be rejected by the Marshall test as unstable for the vertical load indicated although they would prove entirely adequate based on the triaxial values of c and Ø shown.

3. The Marshall stability value may or may not be effected by changes in composition of bituminous mixtures which are reflected by large variations in the c and Ø values.

4. The Marshall stability values are not indicative of the pavement stability under field conditions.

The first two assumptions are not supported by any comparative laboratory data showing the actual Marshall values of the mixtures which have the triaxial values of c and Ø indicated. Until Mr. McLeod submits the composition of the paving mixtures and both the Marshall and triaxial values for each mixture used in Figure 44 it must be considered in connection with the first two assumptions that

1. The inadequacy of the Marshall machine is not proven by Figure 44.

2. Figure 44 is purely a theoretical chart in which assumptions are used as a basis to obtain conclusions which may not be valid or technically correct.

A digest of the investigational report[8] will disclose that the third assumption is not valid as the Marshall stability values reflects the variations of a paving mixture in a similar manner as do the values for c and Ø in the triaxial test. The investigational report[9] contains the laboratory data on the pertinent variations normally found to exist in asphaltic concrete pavements. These variations are as follows:

1. Size, angularity and type of coarse aggregate.
2. Percentage of coarse aggregate.
3. Percentage and penetration of asphalt.
4. Percentage and character of mineral filler.
5. Angularity of fine aggregate.
6. Gradation of aggregate (both fine and coarse).
7. Mixing temperatures.

It was recognized in the Corps of Engineers investigation of asphalt paving mixtures that procedures of preparing test specimens of paving mixtures must be developed so their physical

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[8] Corps of Engineers report, op. cit.

[9] Ibid.

characteristics could be comparable to the field pavements. The present compactive effort used to prepare specimens for the Marshall machine was based on laboratory and rolling studies, the effect of traffic using typical airplane wheel assemblies on pavements of varying composition and the increase in pavement density due to traffic. In addition criteria for pavements based on Marshall stability values and other characteristics suitable for the very heavy wheel loads used were established with due consideration given to field conditions. The supporting data is contained in the investigational report.[10] In view of the above it is considered that the fourth assumption is entirely invalid.

## SUMMARY

In connection with the triaxial tests, the Triaxial Institute recognizes the need for further investigational work on the items listed below and a program for the accomplishment of the necessary work has been initiated.

1. For improvement in preparation of test specimens of paving mixtures so they will have physical characteristics comparable to finished pavements.
2. For adequate correlation of paving mixtures designed by the triaxial test with performance under traffic.
3. For interpretation of the triaxial method of design into simple terms suitable for use by the practicing engineer and laboratory technician.
4. For bringing the triaxial test in as complete correlation as possible with other tests being currently used for design of bituminous pavement.

Past history indicates that the Hubbard-Field, Marshall and Hveem testing machines have been successfully used by design engineers for a considerable period of time in designing adequate bituminous pavement for highways, city streets and airfields.

Mr. McLeod fails to furnish the comparative laboratory Marshall and triaxial values of the paving mixtures used in Figure 44 on which an attempt is made to prove the inadequacy of the Marshall machine for measuring the stability of bituminous paving mixtures.

The preparation of test specimens which have physical characteristics comparable with actual pavements and correlation of pavement performance with test values has been accomplished with regards to the Marshall machine. These important problems are now being investigated by the Triaxial Institute in connection

[10] Ibid.

with the triaxial test which Mr. McLeod bases his rational approach to the design of bitiminous paving mixtures.

Mr. McLeod earnestly advocates the use of a test not fully developed and presents charts and equations based on many assumed and undetermined values.

Further, the agencies who are planning to conduct the investigational work necessary to determine exact values for the numerous factors used in the various charts and equations together with the time required for accomplishment are not indicated in the paper.

The results of investigational work shown on Figure 46 indicates that Marshall stability values measures internal friction.

The method of determining the most suitable asphalt content for a given mixture and design condition by use of the triaxial test is not described in the paper.

The rational approach to design of bituminous pavements does not provide for a factor of safety to compensate for normal deficiencies which occur in the preparation and laying of paving mixtures.

## CONCLUSIONS

It is reasonable to assume that the development of a practical method for the design of bituminous pavements based on triaxial testing adequately correlated with field conditions will result eventually from the work planned by the Triaxial Institute.

It is also reasonable to assume that engineering organizations concerned with design of bituminous pavements will give due consideration to a design method developed by the Triaxial Institute as its personnel is composed of able investigators, competent highway engineers, university professors and representatives of the asphalt industry thoroughly experienced in pavement design.

It is entirely possible that the design equations and figures based on factors with assumed or unknown values will be either invalid or technically incorrect when the values of the factors used have been determined by laboratory and field work.

The criticism that the Marshall stability value is largely a measurement of cohesion, is not indicative of pavement performance and does not reflect changes in composition in a mixture is not borne out by laboratory and field studies conducted by the Corps of Engineers.

As many assumptions unsupported by experimental data have been introduced in Figure 44, the deductions used to prove the inadequacy of the Marshall test for measuring the stability of bituminous paving mixtures must be considered invalid.

## CLOSURE

In view of the above discussion the rational approach to the design of bituminous paving mixtures proposed by Mr. McLeod must be considered only a highly theoretical and academic approach the validity of which depends upon the results of extensive laboratory work and field correlation.

It is considered that before any method either empirical or theoretical can be used with a confidence for designing economical asphalt pavements adequate for heavy traffic correlation of the effect of actual traffic on pavements designed by the method is necessary.

DR. McLEOD (by letter): Mr. Ricketts' remarks are welcomed and appreciated, since the various points of view presented in the section devoted to discussion are very often of greater interest than the paper itself. In addition, the comments and replies frequently lead to a better understanding of the subject matter of the paper itself.

It is not clear to us why, in his opening paragraph, Mr. Ricketts attempts to create the impression that there is some close connection between the subject matter of our paper and that of two recent papers by V. R. Smith, and that the triaxial test procedure and equipment described in our paper is similar to that recommended by Mr. Smith. To anyone who makes even a casual study of the papers by Mr. Smith and by the author, it must be perfectly clear that although both are based upon the triaxial test the approaches to the problem of bituminous pavement design described by Mr. Smith and the author are fundamentally different.

Mr. Smith has very clearly indicated that his method of bituminous mixture design is based upon the theory of elasticity. This is not true of the subject matter for the author's paper. Mr. Smith recommends the testing of bituminous mixtures at zero rate of strain, since this avoids the viscous resistance factor. Our paper, on the other hand, recommends that the rate of strain adopted should bear some relationship to the nature of the stresses imposed by traffic, that is, whether the critical wheel load is to be stationary, moving at a uniform rate of speed, etc., and may, therefore, be variable. Like all other present methods of which we are aware, Mr. Smith's design procedure is concerned only with the paving mixture itself. Our approach, on the other hand, indicates that this does not go far enough, since there are a number of important factors influencing the stability developed by bituminous pavements in service, that are not brought into play until after the pavement has been laid and is in actual use. These factors must therefore be incorporated into any rational method of design. These additional factors include pavement thickness, frictional resistance between pavement and tire and between pavement and base, stability in the direction of the longitudinal versus the transverse axis of the tire contact area, shape of the curve of pressure distribution on the contact area, etc. Mr. Smith has done some excellent work with the triaxial test and it is a pleasure to pay tribute to the fine contribution he has made towards a better understanding of pavement design. Nevertheless, it should be quite apparent that the approach to the design of bituminous mixtures described by Mr. Smith is entirely different from that presented in the author's paper.

Near the beginning of his discussion Mr. Ricketts lists two main reasons for commenting on our paper, the first of which is:

> "To point out that many of the charts and equations are based on a number of assumed design factors with values which are yet to be determined by laboratory and field work."

The same theme is reiterated elsewhere in Mr. Ricketts' discussion.

The author believes that quite adequate precautions were taken throughout the paper itself to avoid any misunderstanding in this respect.

With several exceptions, the diagrams included in our paper are not intended to be final design charts. Their major purpose is to illustrate graphically the important influence that such variables as K, n, f, g, P, Q, t, etc. may have on the stability of a bituminous pavement in the field, and to demonstrate the need for obtaining more information concerning them. Figure 17, for example, indicates the tremendous influence that different magnitudes of the value of K (the factor by which the unconfined compressive strength of a bituminous mixture must be multiplied to give the amount of lateral support provided by the pavement surrounding the loaded area) may have on bituminous pavement design, and why it would be to our advantage to be able to evaluate it with reasonable accuracy, or, even more important, to learn how its value might be made as large as possible compatible with the other desirable characteristics of paving mixtures. Similarly, Figures 28 and 36 demonstrate the advantage of developing reasonably high $f + g$ values (f and g being the coefficients of friction between pavement and tire and between pavement and base, respectively) for pavements subject to stationary wheel loads, or to wheel loads moving at a uniform rate of speed, and Figure 42 indicates the desirability of low $f - g$ values for pavements exposed to severe braking and acceleration stresses. Even more important is the demonstrated advantage of developing a high coefficient of friction (or strong bond) between pavement and base course, since this materially contributes to higher pavement stability under all conditions of wheel loading, stationary, moving at uniform speed, and braking or acceleration stresses. Therefore, the paper points out the desirability and the need for undertaking investigations for determining the conditions required for developing a high frictional resistance or strong bond between bituminous pavements and the many different types of base courses on which they are laid.

It is clearly recognized that not enough is known about the actual magnitude of some of these variables at the present time to make adequate or maximum use of them in the design of bituminous pavements. Considerable laboratory and field investigation will be necessary to provide average values and the range of values possible for each of these different factors. In the meantime, it is instructive to derive the mathematical equations that include these different variables. By means of graphs based upon these equations, the influence of different values for each factor can be usefully demonstrated, and a reasonable estimate of its possible importance made.

We doubt that anyone can read through the section on "Discussion of Stability Criteria for Bituminous Pavement Design" in our paper, and not become aware of the fact that much investigational work must yet be done to evaluate the different factors that seem to have an important bearing on the stability of bituminous pavements. Until more is known about the actual influence of each of these variables on bituminous pavement design, sound engineering requires that they be employed very conservatively. It was with this in mind, that the last paragraph included under the sub-heading "Discussion of Stability Criteria for Bituminous Pavement Design" was included in our paper. This paragraph reads as follows:

> "Finally, until there has been an opportunity to build up information on the field performance of bituminous mixtures designed by the rational method and employing the triaxial test, it would seem prudent to base the design of bituminous mixtures on the stationary load condition, for the maximum pressure applied to the contact area[15,16] and possibly including an impact factor.[24] <u>If particular care is taken to obtain a strong bond between pavement and base</u>, it is believed that the curves of Figure 14, for which K = 1, would provide a conservative basis of design. Later, as confidence in this method of design

becomes established, and as more experimental data become available, concerning average values and the range of values possible for each of the important variables, the refinements indicated in the paper, particularly with regard to Figures 17, 27, 28, 30, 35, 36, 37, 39, 41, 42, and 43, which might lead to a less conservative design for both static and moving loads, could be gradually adopted."

It is believed that this quite adequately answers the several comments made by Mr. Ricketts in this respect.

The author has lacked the laboratory equipment and personnel required to obtain the considerable experimental data needed for the complete documentation of every step in the development presented in our paper. Nevertheless, we cannot agree with the statement appearing in Mr. Ricketts' closing comments, "the rational approach to the design of bituminous paving mixtures proposed by Mr. McLeod must be considered only a highly theoretical and academic approach, the validity of which depends upon the results of extensive laboratory work and field correlation."

It is quite true that considerable laboratory investigation and field correlation are required to confirm any rational method that may be proposed. However, we cannot agree that the method outlined in our paper is as "highly theoretical and academic" as Mr. Ricketts would apparently like the reader to believe. Does Mr. Ricketts suggest, for example, that the very marked and experimentally proven influence of the rate of strain on the stability measured for a bituminous mixture, as demonstrated in Figures 18 and 19, is "highly theoretical and academic?" Very few properties of bituminous mixtures are of more practical significance in providing a better fundamental understanding of the variations in pavement performance under different traffic conditions, and particularly under stationary versus moving wheel loads.

As clearly indicated in the paper itself, the development presented is based essentially on the Mohr diagram, the Coulomb equation, the influence of rates of strain or rates of loading, and the balancing of applied stresses and resistances. The last of these is simply a problem in engineering mechanics. The Mohr diagram and Coulomb equation have been employed as highly useful tools for a considerable number of years to obtain practical solutions to soil mechanics' problems. The stability of bituminous pavements appears to be a special problem in soil mechanics. Consequently, we cannot follow the line of reasoning that leads Mr. Ricketts to conclude that principles, recognized to be of the greatest practical value for the solution of stability problems in the field of soil mechanics, should suddenly become "theoretical and academic," when utilized to solve the problem of the stability of bituminous pavements.

As has just been pointed out, to a very considerable degree, the development presented in our paper examines the stability of bituminous pavements as a problem in applied mechanics. The validity of this development depends upon whether or not the conclusions derived from it are in keeping with practical field observations. In the writer's opinion they very definitely are, and frequent examples of this have been referred to in the paper itself. For example, this development clearly indicates that for stationary wheel loads, or for wheel loads moving at a uniform rate of speed, a thick pavement must be designed to have a higher minimum stability than a thin pavement. Mr. Vokac has emphasized in his earlier discussion of our paper, that this is a matter of common field observation.

Mr. Ricketts reviews at some length the objectives of the Triaxial Institute, and the progress it has made to date. He places special emphasis on the Triaxial Institute's realization that test specimens duplicating pavement structure in the field are necessary before a sound method of design for bituminous pavements based upon the triaxial test can be established.

We are in complete accord with the objectives of the Triaxial Institute, and in Item (2), under the heading "General" in our paper, reference is made to the particularly meritorious work of this Institute towards the development of a laboratory compaction procedure that will provide test specimens having the same structure that develops under rolling and traffic in the field.

We would have been more impressed with Mr. Rickett's comments in this connection, if he had somewhere in his discussion stated with equal emphasis that this is also true of the various empirical tests, such as Hubbard-Field, Marshall, and Hveem Stabilometer, in common use at this time. (Mr. Hveem himself, of course, has given this factor the emphasis it deserves.) The development of a method for preparing test specimens that will duplicate pavement structure in the field is just as important in connection with the use of empirical tests as in the case of the triaxial test, and particularly since the results of these laboratory tests must be tied in with field performance, if they are to be of any value. It is not enough to devise a compaction procedure that merely duplicates the density obtained in the pavement in service. It must be able to produce in compacted samples the actual pavement structure developed in the field.

Mr. Ricketts states that "no consideration is given to a factor of safety to provide for vagaries which occur many times during preparation and laying of pavement mixtures." This is a matter that each design engineer would wish to select for himself, and it could be easily incorporated into the design of a bituminous mixture by multiplying the design load by a safety factor. Two references to this procedure for other purposes are contained in the paper itself. The first occurs in the last paragraph of Item (6) under the sub-heading "General," and states "Nevertheless, if in spite of these observations field experience should indicate that repetitions of traffic loads must be considered in connection with the stability of bituminous pavements, they could probably be satisfactorily taken into account by multiplying the design load by a traffic factor. This would be equivalent to employing a safety factor." The second appears in the first sentence of the last paragraph under the sub-heading "Discussion of Stability Criteria for Bituminous Pavement Design," and reads as follows, "Finally, until there has been an opportunity to build up information on the field performance of bituminous mixtures designed by the rational method and employing the triaxial test, it would seem prudent to base the design of bituminous mixtures on the stationary load conditions, for the maximum pressure applied to the contact area,[15,16] and possibly including an impact factor.[24]"

Anyone who studies Figures 33, 34, 35, and 36 of our paper will realize that if the method of design outlined in the second quotation just given were adopted, it would provide a safety factor that could vary from 1 to 10 or more, depending upon the $f + g$ value actually developed by the pavement in place. A safety factor of 10 is, of course, ridiculously high, but it is not unlikely that many bituminous pavements having a safety factor of this order are now actually being laid each year, chiefly because present methods of design based upon empirical tests such as Hubbard-Field, Marshall, and Hveem Stabilometer, are fundamentally incapable of providing the data required to determine the value of the safety factor employed and to ascertain the degree to which the pavement on any given project has been either overdesigned or underdesigned. It is one of the objectives of a rational method of pavement design to indicate the magnitude of the safety factor employed, and to avoid either underdesign or excessive overdesign.

Mr. Ricketts makes the criticism that "the method for determining the most suitable asphalt content for a given mixture to be used for a given design condition by the use of the triaxial test is not described either in the text or the equations and charts." As carefully pointed out in the introduction, this matter was considered to be outside the scope of the present paper, which is concerned only with the determination or strength of a bituminous paving mixture after it has been adequately designed for workability, density, durability, etc. It is widely accepted today, that a bituminous pavement should contain the maximum amount of binder possible, without losing stability in service, and that the air voids at the greatest density it will acquire in service should be not less than from 2 to 3 per cent. The unfluence of the asphalt content on the values of c and Ø measured for an asphalt paving mixture by means of the triaxial test is indicated in papers by Goetz and Chen in the present Proceedings, and in the paper by V. R. Smith presented at the A.A.P.T. annual meeting a year ago.

Concerning our Figure 11, Mr. Ricketts states, "Here again is an assumption, as engineering data is not offered to substantiate whether all mixtures considered satisfactory possess an L value of 30 p.s.i." It should be emphasized in this connection that Figure 11 is simply the graphical solution provided when equation (3) is employed to obtain the answer to a specific problem for which the stress conditions were clearly defined, i.e. V = 100 p.s.i. and L = 50 p.s.i. Figure 11 is intended to apply only to the material within an element carrying these specified stresses under equilibrium conditions. Figure 11 is not concerned with the nature of the material outside the stressed element, except that it must be placed in such a manner as to provide a lateral support L of 30 p.s.i. Consequently, Figure 11 is not intended to deal with the question raised by Mr. Ricketts concerning how much latteral support the material outside the loaded element can provide. This problem, however, is taken up in connection with Figures 12, 13, and 14.

The text of the paper itself clearly points out that equation (3) and corresponding illustrative diagrams like Figure 11 are of little practical value unless some means can be developed for determining the amount of lateral support L provided by the pavement adjacent to the loaded area. A method for this purpose is described in connection with Figures 12, 13, and 14. This method indicates that the unconfined compressive strength of the paving mixture, multiplied by a factor K, provides a reasonable evaluation of this lateral support L. Figure 17 demonstrates the marked influence that different values of K would have on bituminous mixture design. No experimental data concerning the magnitude of the value of this factor K for bituminous pavements in service seem to be available. Text books on soil mechanics, however, show indirectly that a value of K = 1 should ordinarily be conservative. Consequently, any engineer wishing to use the method of design outlined would probably be quite safe in assuming a value for $K = 1$. That is, the assumption that the lateral support L available is equal to the unconfined compressive strength would appear to be conservative.

It is not improbable that the structure of a bituminous pavement is somewhat different in a horizontal than in a vertical direction. If this should be the case, and if it is of sufficient importance, the value of the unconfined compressive strength to be employed for evaluating L might have to be measured by the procedure for running the triaxial test employed by Professor Haefeli at the University of Zurich in Switzerland, in which the vertical load V becomes the minor principal stress, and the horizontal pressure L is the major principal stress.

Mr. Ricketts refers to our Figure 44, which demonstrates the fundamental inadequacy of the unconfined compression test for determining the stability that bituminous pavements develop under service conditions. Insofar as the Marshall stability test is related to the unconfined compression test, that remarks in our paper with reference to Figure 44 apply to it also.

Mr. Ricketts states, "the Marshall stability values reflect the variations of a paving mixture in a similar manner as do the values for c and $\phi$ in the triaxial test." A moment's reflection should indicate that this statement cannot be true. Graphs of Marshall stability values versus bitumen content contained in Highway Research Board Research Report 7-B, Symposium on Asphalt Paving Mixtures, indicate that except at the peak there are two points on each curve where the Marshall stability value is exactly the same, but the asphalt contents are different, one being on the lean side and the other on the rich side of optimum. This provides a very simple and conclusive example, therefore, of variations in the composition of a paving mixture that do not change the Marshall stability value, but would result in different Mohr envelopes as shown in Figure 44 (a).

On page 246, Volume 18, Proceedings of The Association of Asphalt Paving Technologists, Vokac, by means of an isometric chart, indicates the wide range of changes that can be made in the composition of bituminous mixtures and yet maintain a constant value for the unconfined compressive strength. Rice and Goetz provide similar diagrams in the same volume. No one familiar with triaxial testing would suggest that all of these mixtures of widely varying composition, but with a constant value for unconfined compressive strength, could be represented by a single Mohr envelope, that is, by constant values for c and $\phi$. Consequently, although no actual comparative test data are provided in connection with Figure 31, the information obtained by other investigators strongly infers that the relationships shown in Figure 44 between unconfined compression and triaxial tests are correct. Insofar as the Marshall test is related to the unconfined compression test, the relationships indicated in Figure 44 undoubtedly apply to it also.

Mr. Ricketts points out that the Marshall test values vary with changes in aggregate grading, aggregate type, percentage of coarse aggregate, shape and surface texture of aggregate particles, percentage and character of filler, percentage and penetration of bituminous binder, etc. These variations in mixture composition would also cause changes in the unconfined compressive strength corresponding to the changes in Marshall stability. However, Figure 44 was presented for the specific purpose of demonstrating that changes in the unconfined compressive strength of bituminous mixtures do not necessarily indicate that corresponding changes in stability will occur when these paving mixtures are laid in the field. Thus, it could very easily happen that a change in mixture composition would decrease the unconfined compressive strength, but the stability developed by the pavement under field conditions would increase as a result of this change. The reverse could also very easily happen, and clearly demonstrates that the unconfined compressive strength test, regardless of how sensitive it may be to changes in mixture composition, is fundamentally incapable of providing a satisfactory criterion for the stability that a paving mixture will develop under service conditions, when laid as a pavement. On the other hand, Figure 44 clearly indicates that the triaxial test should be capable of indicating the stability to be developed by a pavement in the field.

Mr. Ricketts objects to our advocacy of the triaxial test because he believes that the work of the Triaxial Institute indicates it to be in only the development stage. Can it be safely said of the three empirical tests, Hubbard-Field, Marshall, and Hveem Stabilometer, that they are completely out of the development stage? In connection with the use of the Marshall test, Mr. Ricketts states that the Corps of Engineers is still "continuing investigational work on bituminous paving mixtures to provide pavements adequate for the traffic of all types of newly developed military planes." If wheel loads or tire pressures were changed materially, it would be found that further development work was required in connection with the Hveem stabilometer and Hubbard-Field tests also.

It should be observed in this connection that even The Asphalt Institute recommends the design of asphaltic concrete by means of the triaxial test, in its "Manual on Hot-Mix Asphaltic Concrete

Paving." While this method of design was established on a much different basis than that outlined in our paper, and we believe that it should be modified, nevertheless, the author feels that the triaxial method described by The Asphalt Institute, together with the stability diagram accompanying it, is conservative, and can be safely used for asphalt pavement design for the traffic on highways and city streets, provided always that the bituminous pavement is firmly bonded to the base course.

Mr. Ricketts states that "Mr. McLeod's criticism regarding the inadequacy of existing machines and methods, particularly Hubbard-Field, Marshall, and Hveem, to properly design bituminous pavements is unwarranted and is not substantiated by experience." As pointed out in the paper itself, after having given careful study to this matter, the author is firmly convinced that the Hubbard-Field, Marshall, and Hveem Stabilometer tests are fundamentally incapable of providing a satisfactory measure of the stability of bituminous pavements in service. The author also believes that of all the stability tests with which he is familiar at the present time, the triaxial is the only practical test capable of providing the fundamental data required for this purpose.

This conviction is based upon the simple observation that any one of the present commonly used empirical stability tests, when made on a sample of a bituminous paving mixture in the laboratory, provides a result that may not be indicative of the stability that the same paving mixture will develop when laid as a pavement in the field, and may even be quite misleading in this respect. This is due to the fact that the sample tested by these methods for stability in the laboratory is not subjected to the same conditions during the test to which it is exposed in service after being laid as a pavement on highway, street, or runway.

Every engineer having wide experience with bituminous pavements has observed paving mixtures develop instability in the field, although high stability values had been or would be reported for them in the laboratory. Conversely, he has seen mixtures, for which laboratory tests would show questionable stability, perform without developing any indications of instability in the field. Thus, practical experience has demonstrated the truth of the statement in the previous paragraph, that the stability value reported for a bituminous mixture by an empirical test in the laboratory does not necessarily represent the stability that the same mixture will develop after being laid as a pavement in the field.

On the other basis of the development described in the paper itself, when all other factors are equal, the stability developed by a bituminous pavement in service may depend upon:

(1) the magnitude of the lateral support L provided by the pavement adjacent to the loaded area
(2) pavement thickness
(3) frictional resistance between pavement and tire
(4) frictional resistance between pavement and base
(5) influence of rate of loading or rate of strain on the value of cohesion c, and possibly on the value of the angle of internal friction $\phi$
(6) stability in the direction of the longitudinal versus the transverse axis of the tire contact area
(7) shape of the curve of pressure distribution on the contact area
(8) whether the wheel loading is applied as a stationary load, as a load moving at a relatively uniform rate of speed, or as a load that subjects the pavement to severe braking or acceleration stresses.

The three empirical tests, Hubbard-Field, Marshall, and Hveem stabilometer, are fundamentally incapable of explaining why any one of the eight factors just summarized should have an influence on the stability that a paving mixture develops in service after being laid as a pavement. On the other hand, if the development outlined in our paper is even qualitatively correct, the triaxial test appears to be capable of providing fundamental information that makes it a reasonably simple matter to understand why each one of these eight factors should have a marked influence on the stability that a paving mixture will develop in the field.

Mr. Ricketts refers to the fact that a considerable mileage of pavements for highways, streets and airports have been designed by Hubbard-Field, Marshall, and Hveem tests, and that good correlation between test methods and field performance has been established. One is inclined to enquire just how carefully and thoroughly this correlation has been studied and what price is being paid in terms of unnecessary overdesign where such correlation has been demonstrated. There is very good reason for suspecting that where good correlation has been established, either with or without excessive overdesign, it is not entirely due to the merit of the empirical stability test employed. As has just been pointed out, in addition to the quality of the paving mixture itself, pavement stability is dependent upon a number of other factors that are not brought into play until after the pavement has been laid and is in service. Consequently, where apparent good correlation between empirical test and pavement performance is observed, it may be partly due, for example, to the assistance unwittingly provided by the construction engineer on the job, through his care in obtaining a good bond between pavement and base. There is nothing in the design data obtained from the Hubbard-Field, Marshall, or Hveem Stabilometer tests to indicate that the stability of the finished pavement may depend very materially on good frictional resistance between pavement and base. Nevertheless, construction engineers have instinctively made provision for good bonding between pavement and base through prime coat, tack coat, etc. We suspect that in locations where this bonding of pavement to base is weak or indifferent, the correlation between the stability values provided by these empirical tests and the field performance leaves considerable to be desired. The triaxial test, on the other hand, provides data from which, as shown in the paper, the important influence of a good bond between pavement and base on pavement stability can be easily demonstrated. The clear recognition of the importance of this fact alone could lead to considerable economy in bituminous pavement design and construction.

A serious disadvantage of the use of empirical tests is that the actual degree of overdesign or underdesign of a bituminous pavement is never known. In most engineering fields safety factors are used, but from the laws of the strength of the materials employed, a reasonable estimate of the safety factors can be made. This cannot be done in connection with our bituminous pavements as long as we continue to rely on the present empirical tests. The development of a rational method of design would give some control over the safety factor employed. This, in turn, should lead to worthwhile pavement economy, because of the greater confidence in local materials, and the wider selection of aggregates, it should make possible.

It should be pointed out that the author is not alone in his criticism of our currently used empirical tests. A concise review of the short-comings of empirical tests is contained in Nijboer's book "Plasticity as a Factor in the Design of Dense Bituminous Road Carpets." Smith has also criticized some of our present empirical tests in his paper for Vol. 18 of the A.A.P.T. Proceedings and in his discussion in Highway Research Board Research Report 7-B, "Symposium on Asphalt Paving Mixtures."

In the summary of his discussion, Mr. Ricketts makes the comment that "the agencies who are planning to conduct the investigational work necessary to determine exact values for the numerous factors used in the various charts and equations together with the time required for accomplishment are not given in the paper." We had considered this matter to be quite outside the scope of our paper, but we are glad to comment briefly on Mr.

Ricketts' remark. The necessary investigational work will be carried out chiefly by those organizations and individuals, who, like the author, feel that we have now reached the stage where a serious effort should be made to emancipate the design of bituminous pavements from its dominance by the current empirical tests, and to establish it on a sound rational basis. Endersby, Smith, Nijboer, Goetz and Chen, The Triaxial Institute, and others have already made important contributions in this direction. It might be added that the organization to which Mr. Ricketts belongs would profit as much as any from the development of a rational method of design for bituminous paving mixtures.

As has happened for a period of time in the early stages in nearly every indsutry, the art of designing bituminous pavements has been ahead of the science, and it has been inevitable that empirical tests should have been developed in the past in an endeavour to measure their stability. As has also frequently occurred in other scientific fields, the author believes we have reached the stage where further adherence to these empirical tests, and to the outmoded ideas they represent, threatens to become a serious road block on the path of progress in this particular field. Whether the present empirical tests can be correlated with any rational method of design ultimately developed, or whether one or more entirely new rapidly performed tests for purposes of control that can be so correlated will have to be devised, is for the future to answer. Be that as it may, the author believes that the time has come when serious consideration should be given to developing a rational method of bituminous pavement design, and to breaking away from our present dependence on empirical tests. The author's paper represents one attempt to make this break from the previous well-beaten path and there will be others. It is fully realized that further refinements to the development we have described may be required, and that there may be quite other approaches to the rational design of bituminous mixtures that could provide equally accurate solutions. It is inevitable that there should be a continued search for these.

As long as we maintain empirical tests as our only basis for the design of bituminous paving mixtures, whenever a wheel loading or tire pressure changes materially, we are faced with another long, costly, laborious cycle of laboratory testing and field correlation, which, in the end, is not necessarily reliable. In this respect, our present empirical approach to bituminous pavement design is not far removed from the engineering technique of the ancients, who had to load a newly completed structure to failure, when it is necessary to determine what weight it could safely carry.